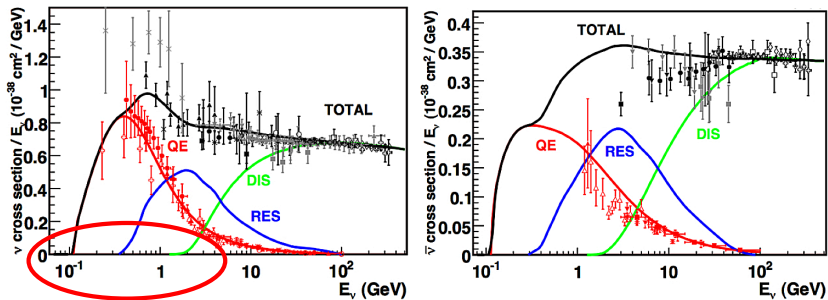


Short-range correlations in neutrino-nucleus scattering

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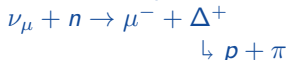
ESTN workshop - Neutrino-nucleus scattering,
Apr 20, 2016



- ▶ QE - Quasi-elastic scattering: nucleon stays intact

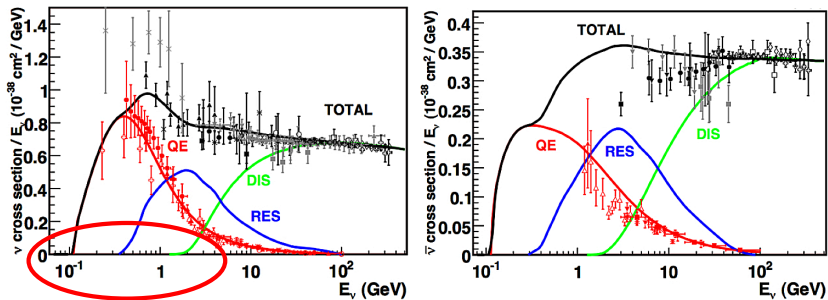


- ▶ RES - Resonance production: nucleon is excited



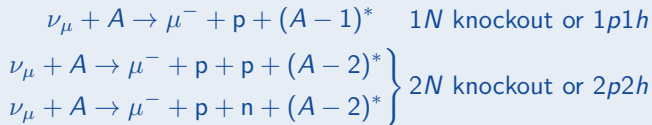
- ▶ DIS - Deep inelastic scattering: nucleon breaks up



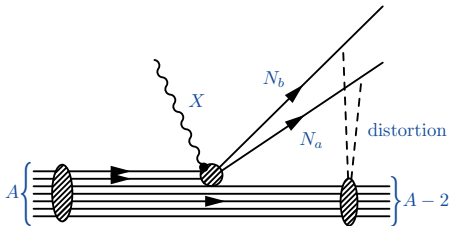
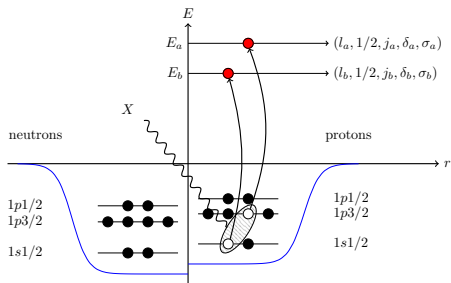


Multinucleon effects

Dip region: multinucleon effects necessary to explain data



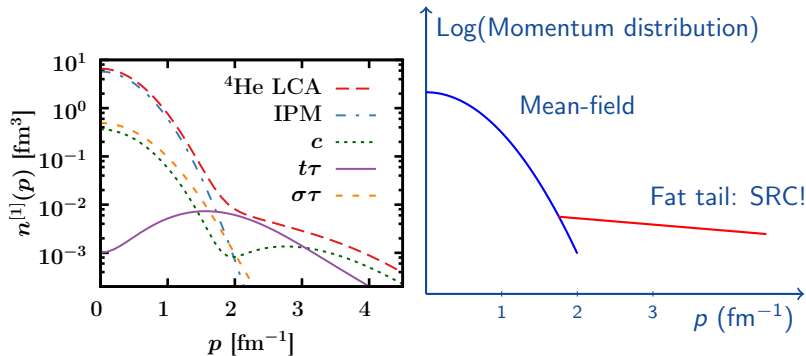
Nuclear model



- ▶ Ground state nucleus is a **shell model**
 - ▶ Calculated with a Hartree-Fock approximation using an effective Skyrme NN force (SkE2)
 - ▶ Accounts for binding energies and nuclear structure
 - ▶ Pauli-blocking effects included inherently
- ▶ Continuum wave functions are calculated using the **same NN potential**
 - ▶ Orthogonality is preserved between initial and final states
 - ▶ Distortion effects of the residual nucleus on the ejected nucleons are incorporated

Short-range correlations

Fat tails in the single-nucleon momentum distribution cannot be explained using an IPM (see talk J. Ryckebusch)

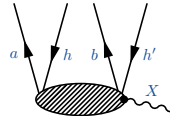
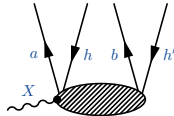
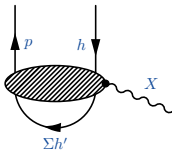


Ref: J. Ryckebusch, *et al.*, J. Phys. G: Nucl. Part. Phys. 42 055104 (2015)

Short-range correlations

Fat tails in the single-nucleon momentum distribution cannot be explained using an IPM (see talk J. Ryckebusch)

- ▶ Nucleons occur in pairs with high relative momenta and low center-of-mass momenta (SRC pairs)
 - ▶ tensor correlations dominate at intermediate relative pair momenta
 - ▶ central correlations dominate at high relative pair momenta
- ▶ A signature of SRC is **back-to-back** $2N$ knockout
- ▶ SRC also have an effect on $1N$ knockout



Ref: J. Ryckebusch, *et al.*, Nucl.Phys. A624, 581 (1997)
S. Janssen, *et al.*, Nucl.Phys. A672, 285 (2000)
(electron-scattering model with SRC + MEC)

Short-range correlations

Correlated wave functions $|\Psi\rangle$ are constructed by acting with a many-body correlation operator $\hat{\mathcal{G}}$ on the uncorrelated Hartree-Fock wave functions $|\Phi\rangle$

$$|\Psi\rangle = \frac{1}{\sqrt{\mathcal{N}}} \hat{\mathcal{G}} |\Phi\rangle, \quad \text{with} \quad \mathcal{N} = \langle \Phi | \hat{\mathcal{G}}^\dagger \hat{\mathcal{G}} | \Phi \rangle$$

The central (c), tensor ($t\tau$) and spin-isospin ($\sigma\tau$) correlations are responsible for majority of the strength

$$\hat{\mathcal{G}} \approx \hat{\mathcal{S}} \left(\prod_{i < j}^A [1 + \hat{l}(i, j)] \right)$$

with $\hat{\mathcal{S}}$ the symmetrization operator and

$$\hat{l}(i, j) = -g_c(r_{ij}) + f_{t\tau}(r_{ij}) \hat{\mathcal{S}}_{ij} (\boldsymbol{\tau}_i \cdot \boldsymbol{\tau}_j) + f_{\sigma\tau}(r_{ij}) (\boldsymbol{\sigma}_i \cdot \boldsymbol{\sigma}_j) (\boldsymbol{\tau}_i \cdot \boldsymbol{\tau}_j).$$

$g_c(r_{ij})$, $f_{t\tau}(r_{ij})$ and $f_{\sigma\tau}(r_{ij})$ are the respective correlation functions

Short-range correlations

Transition matrix elements between **correlated states** $|\Psi\rangle$ can be written as matrix between **uncorrelated states** $|\Phi\rangle$, with an effective transition operator

$$\langle \Psi_f | \hat{J}_\mu^{\text{nucl}} | \Psi_i \rangle = \frac{1}{\sqrt{\mathcal{N}_i \mathcal{N}_f}} \langle \Phi_f | \hat{J}_\mu^{\text{eff}} | \Phi_i \rangle,$$

with

$$\hat{J}_\mu^{\text{eff}} = \hat{\mathcal{G}}^\dagger \hat{J}_\mu^{\text{nucl}} \hat{\mathcal{G}} = \left(\prod_{j < k}^A [1 + \hat{l}(j, k)] \right)^\dagger \hat{J}_\mu^{\text{nucl}} \left(\prod_{l < m}^A [1 + \hat{l}(l, m)] \right).$$

In the IA, the many-body nuclear current can be written as a sum of one-body operators

$$\hat{J}_\lambda^{\text{eff}} = \left(\prod_{j < k}^A [1 + \hat{l}(j, k)] \right)^\dagger \sum_{i=1}^A \hat{J}_\lambda^{[1]}(i) \left(\prod_{l < m}^A [1 + \hat{l}(l, m)] \right).$$

Short-range correlations

Use the fact that SRC is a **short-range** phenomenon

- ▶ Terms linear in the correlation operator are retained
- ▶ A -body operator \rightarrow 2-body operator

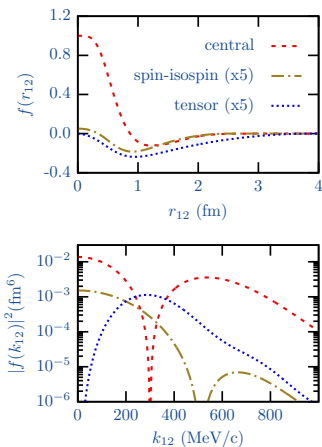
$$\hat{J}_\lambda^{\text{eff}} \approx \underbrace{\sum_{i=1}^A \hat{J}_\lambda^{[1]}(i)}_{\text{one-body (IA)}} + \underbrace{\sum_{i < j}^A \hat{J}_\lambda^{[1],\text{in}}(i,j)}_{\text{two-body (SRC)}} + \left[\sum_{i < j}^A \hat{J}_\lambda^{[1],\text{in}}(i,j) \right]^\dagger$$

where

$$\hat{J}_\lambda^{[1],\text{in}}(i,j) = \left[\hat{J}_\lambda^{[1]}(i) + \hat{J}_\lambda^{[1]}(j) \right] \hat{l}(i,j)$$

- ▶ Effective nuclear current is the sum of a one-body (IA) and two-body (SRC) current

Short-range correlations



- ▶ The correlations have a **short range**:
 $f(r_{ij}) \rightarrow 0$ at $r_{ij} > 3$ fm
- ▶ Tensor correlation function dominates for intermediate relative momenta 200 – 400 MeV/c
- ▶ Central correlation function dominates at high relative momenta
- ▶ Spin-isospin correlation function overall relatively small
- ▶ These correlation functions are input, obtained from literature

Ref: C. C. Gearhaert, PhD thesis, Washington University, (1994). (central)
S. C. Pieper, *et al.* Phys.Rev. C46, 1741 (1992). (tensor and spin-isospin)

Figure: Correlation functions

One-nucleon knockout

Directly calculate the double differential cross section

$$\frac{d\sigma}{dE_{e'} d\Omega_{e'}} = 4\pi\sigma^X \zeta f_{rec}^{-1} [v_{CC} W_{CC} + v_{CL} W_{CL} + v_{LL} W_{LL} + v_T W_T - h v_{T'} W_{T'}],$$

with v and σ^X containing leptonic information, e.g.

$$\sigma^{\text{Mott}} = \left(\frac{\alpha \cos(\theta_{e'}/2)}{2E_e \sin^2(\theta_{e'}/2)} \right)^2, \quad \sigma^W = \left(\frac{G_F \cos\theta_c E_\mu}{2\pi} \right)^2,$$

and the response functions containing the nuclear information

$$W_{CC} = |\mathcal{J}_0|^2$$

$$W_{CL} = 2 \text{Re} \left(\mathcal{J}_0 \mathcal{J}_3^\dagger \right)$$

$$W_{LL} = |\mathcal{J}_3|^2$$

$$W_T = |\mathcal{J}_+|^2 + |\mathcal{J}_-|^2$$

$$W_{T'} = |\mathcal{J}_+|^2 - |\mathcal{J}_-|^2$$

$$\mathcal{J}_0 = \langle \Psi_f | \hat{J}_0(q) | \Psi_i \rangle$$

$$\mathcal{J}_+ = \langle \Psi_f | \hat{J}_+(q) | \Psi_i \rangle$$

$$\mathcal{J}_- = \langle \Psi_f | \hat{J}_-(q) | \Psi_i \rangle$$

$$\mathcal{J}_3 = \langle \Psi_f | \hat{J}_3(q) | \Psi_i \rangle$$

SRC results - $1p1h$

The effective two-body operator affects the $1p1h$ cross section

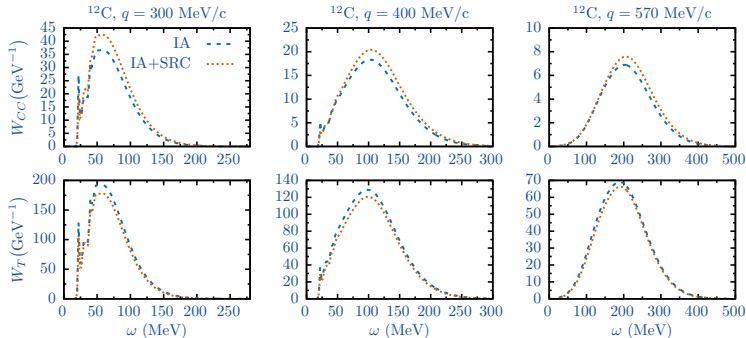


Figure: W_{CC} and W_T response functions for $1p1h$ $^{12}\text{C}(\nu_\mu, \mu^-)$

- ▶ Small increase in longitudinal channel W_{CC}
- ▶ Small decrease in transverse channel W_T

Two-nucleon knockout

Start with the 8-fold exclusive differential cross section

$$\frac{d\sigma}{dE_f' d\Omega_f' dT_a d\Omega_a d\Omega_b} = \sigma^X \zeta f_{rec}^{-1} \\ \times \left[v_{CC} W_{CC} + v_{CL} W_{CL} + v_{LL} W_{LL} + v_T W_T + v_{TT} W_{TT} + v_{TC} W_{TC} \right. \\ \left. + v_{TL} W_{TL} - h(v_{T'} W_{T'} + v_{TC'} W_{TC'} + v_{TL'} W_{TL'}) \right],$$

The leptonic factors v and σ^X are independent of the number of knockout particles and five more response functions appear

$$W_{TT} = 2 \operatorname{Re} \left(\mathcal{J}_+ \mathcal{J}_-^\dagger \right)$$

$$W_{TC} = 2 \operatorname{Re} \left(\mathcal{J}_0 (\mathcal{J}_+ - \mathcal{J}_-)^{\dagger} \right)$$

$$W_{TL} = 2 \operatorname{Re} \left(\mathcal{J}_3 (\mathcal{J}_+ - \mathcal{J}_-)^{\dagger} \right)$$

$$W_{TC'} = 2 \operatorname{Re} \left(\mathcal{J}_0 (\mathcal{J}_+ + \mathcal{J}_-)^{\dagger} \right)$$

$$W_{TL'} = 2 \operatorname{Re} \left(\mathcal{J}_3 (\mathcal{J}_+ + \mathcal{J}_-)^{\dagger} \right)$$

$$\mathcal{J}_0 = \langle \Psi_f | \hat{J}_0(q) | \Psi_i \rangle$$

$$\mathcal{J}_+ = \langle \Psi_f | \hat{J}_+(q) | \Psi_i \rangle$$

$$\mathcal{J}_- = \langle \Psi_f | \hat{J}_-(q) | \Psi_i \rangle$$

$$\mathcal{J}_3 = \langle \Psi_f | \hat{J}_3(q) | \Psi_i \rangle$$

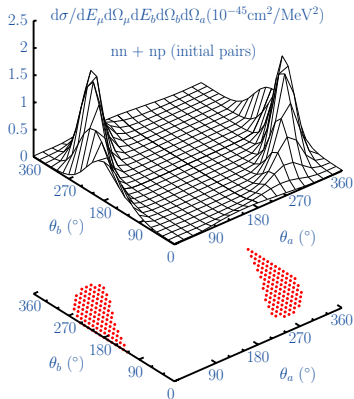
Adding $2p2h$ cross section to $1p1h$ double differential cross section
 \rightarrow integrate over outgoing nucleons $\int dT_a d\Omega_a d\Omega_b$

SRC results - Exclusive $A(\nu_{\mu}, \mu^{-} N_a N_b)$

Exclusive

- ▶ 2 outgoing nucleons N_a and N_b observed in coincidence with the final lepton
- ▶ incoherent sum of pp' and pn knockout
- ▶ $2N$ knockout from all possible shell combinations $(1s1/2)^2$, $(1s1/2)(1p3/2)$ and $(1p3/2)^2$

SRC results - Exclusive $A(\nu_\mu, \mu^- N_a N_b)$



$$\frac{d\sigma}{dE_\mu d\Omega_\mu dT_b d\Omega_b d\Omega_a}(\nu_\mu, \mu^- N_a N_b)$$

$$N_a = p, N_b = p', n$$

- ▶ exclusive differential cross section shows clear back-to-back knockout signal: use this to calculate some of the integrals analytically

Figure: $E_{\nu_\mu} = 750 \text{ MeV}$, $E_\mu = 550 \text{ MeV}$, $\theta_\mu = 15^\circ$ and $T_p = 50 \text{ MeV}$ in lepton scattering plane ($\varphi_a, \varphi_b = 0^\circ$) on ^{12}C .

- ▶ **red area:** $P_{12} = \mathbf{p}_a + \mathbf{p}_b - \mathbf{q} < 300 \text{ MeV}/c$
- ▶ $2N$ knockout cross section proportional to close proximity pairs (see talks J. Ryckebusch and C. Colle)

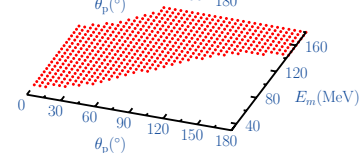
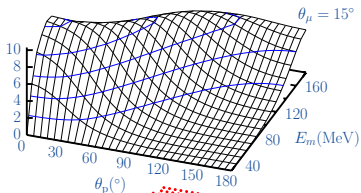
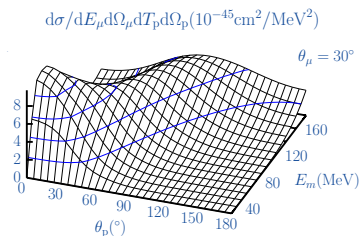
SRC results - Semi-exclusive $A(\nu_{\mu}, \mu^{-} N_a)$

Semi-exclusive (semi-inclusive?)

- ▶ 1 outgoing nucleon N_a observed in coincidence with final lepton
- ▶ $(A - 1)^*$ excited above $2N$ emission threshold
- ▶ contribution of $2N$ knockout $A(l, l' N_a N_b)$ to semi-exclusive $A(l, l' N_a)$
- ▶ incoherent sum of pp' and pn knockout (p is detected)
- ▶ $2N$ knockout from all possible shell combinations
 $(1s1/2)^2$, $(1s1/2)(1p3/2)$ and $(1p3/2)^2$

$$\begin{aligned} \frac{d\sigma}{dE_{l'} d\Omega_{l'} dT_p d\Omega_p} (l, l' p) &= \int d\Omega_n \frac{d\sigma}{dE_{l'} d\Omega_{l'} dT_p d\Omega_p d\Omega_n} (l, l' pn) \\ &+ \int d\Omega_{p'} \frac{d\sigma}{dE_{l'} d\Omega_{l'} dT_p d\Omega_p d\Omega_{p'}} (l, l' pp') \end{aligned}$$

SRC results - Semi-exclusive $A(\nu_\mu, \mu^- N_a)$



$$\int d\Omega_n \frac{d\sigma}{dE_{l'} d\Omega_{l'} dT_p d\Omega_p d\Omega_n}(l, l' pn)$$

We impose quasi-deuteron kinematics and replace $\mathbf{p}_b \rightarrow \mathbf{p}_b^{ave}$

$$\mathbf{p}_b^{ave} = \mathbf{q} - \mathbf{p}_a$$

This is equivalent with residual nucleus with zero recoil momentum ($f_{rec} = 1$)

Figure: $E_{\nu_\mu} = 750$ MeV, $E_\mu = 550$ MeV for in-plane kinematics ($\varphi_p = 0^\circ$) on ^{12}C .

► **red area:** so-called ridge

$$E_m = \frac{A-2}{A-1} \frac{p_m^2}{2m_N} + S_{2N} + E_{A-2}^{hh'}$$

SRC results - Inclusive $2p2h$ $A(\nu_\mu, \mu^-)$

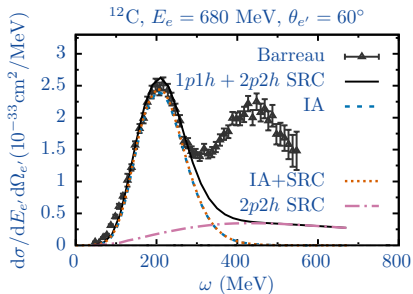
Inclusive $2p2h$

- ▶ only final lepton is detected
- ▶ contribution of $2N$ knockout $A(l, l' N_a N_b)$ to $A(l, l')$
- ▶ incoherent sum of pp' and pn knockout
- ▶ $2N$ knockout from all possible shell combinations $(1s1/2)^2$, $(1s1/2)(1p3/2)$ and $(1p3/2)^2$

$$\frac{d\sigma}{dE_{l'} d\Omega_{l'}}(l, l') = \int dT_p d\Omega_p \frac{d\sigma}{dE_{l'} d\Omega_{l'} dT_p d\Omega_p}(l, l' p)$$

- ▶ $\int d\Omega_p$ analytical integration
- ▶ $\int dT_p$ numerical integration

SRC electron results - Inclusive $2p2h$



Strength of the $2p2h$ contribution

- ▶ tensor SRC dominates at small to intermediate ω
- ▶ central SRC dominates at large ω
- ▶ tensor dominated by pn pairs
- ▶ vector, axial and interference terms are equally important

Figure: (e, e') scattering on ^{12}C

- ▶ Inclusion of SRC in the $2p2h$ channel yields a broad background over the whole ω range

SRC results - Inclusive $2p2h$

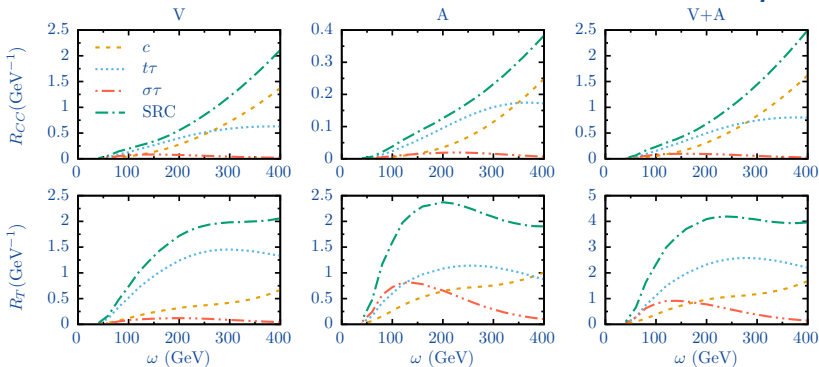


Figure: $2p2h$ SRC response functions R_{CC} and R_T for $^{12}\text{C}(\nu_\mu, \mu^-)$ at $q = 400$ MeV/c

- ▶ The tensor part yields the biggest contribution for small ω transfers while the importance of the central part increases with ω . This is directly related to the correlation functions.
- ▶ Spin-isospin is relatively large for axial-transverse. Axial-transverse current and spin-isospin operator have a $\sigma \cdot \tau$ structure.

SRC results - Inclusive $2p2h$

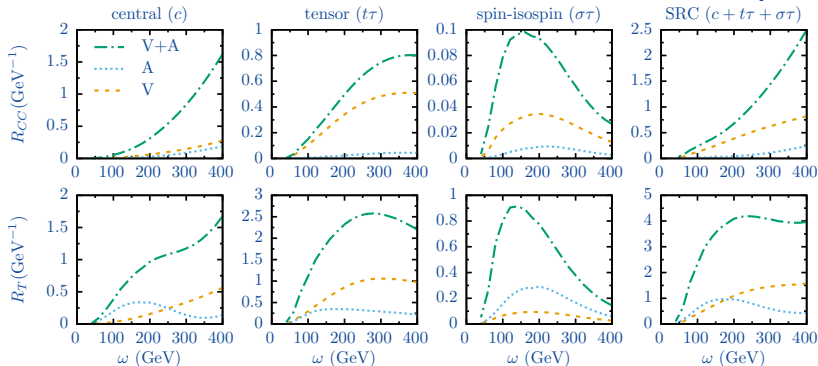


Figure: $2p2h$ SRC response functions R_{CC} and R_T for $^{12}\text{C}(\nu_\mu, \mu^-)$ at $q = 400$ MeV/c

- Vector and axial-vector are both equally important.

SRC results - Inclusive $2p2h$

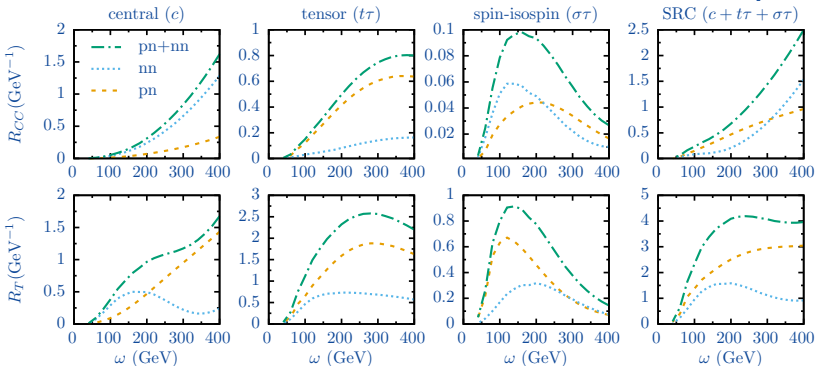
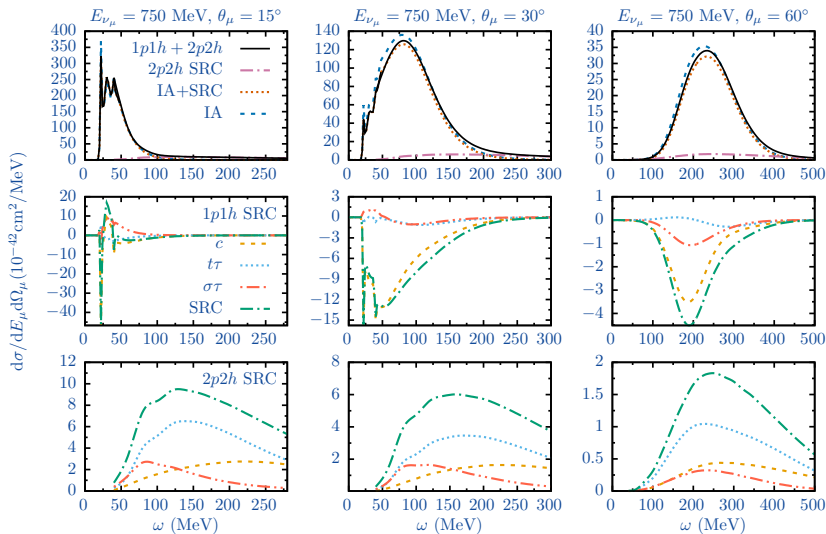


Figure: $2p2h$ SRC response functions R_{CC} and R_T for $^{12}\text{C}(\nu_\mu, \mu^-)$ at $q = 400$ MeV/c

- ▶ Central-Coulomb (top left) does not distinguish between protons and neutrons \rightarrow biggest contribution from nn pairs.
- ▶ Tensor correlations clearly dominated pn pairs.

SRC results - Inclusive $2p2h$



► Inclusive $2p2h$ appears as a broad background to $1p1h$

Meson-exchange currents

Extend the current model with MEC. First seagull and pion-in-flight currents

$$\hat{J}_\lambda^{\text{eff}} = \underbrace{\sum_{i=1}^A \hat{J}_\lambda^{[1]}(i)}_{\text{one-body (IA)}} + \underbrace{\sum_{i<j}^A \hat{J}_\lambda^{[1],\text{in}}(i,j) + \left[\sum_{i<j}^A \hat{J}_\lambda^{[1],\text{in}}(i,j) \right]^\dagger}_{\text{two-body (SRC)}} + \underbrace{\sum_{i<j}^a \hat{J}_\lambda^{[2],\text{sea}}(i,j) + \sum_{i<j}^a \hat{J}_\lambda^{[2],\text{pif}}(i,j)}_{\text{two-body (MEC)}}$$

- ▶ Effective current includes SRCs and MECs in a uniform way
- ▶ Interference between IA, SRC and MEC inherently included

Meson-exchange currents

Vector seagull and pion-in-flight currents

$$\hat{J}_V^{[2],\text{sea,nr}}(\mathbf{q}) = -i \left(\frac{f_{\pi NN}}{m_\pi} \right)^2 (\vec{\tau}_1 \times \vec{\tau}_2)_\pm F_1^V(Q^2) \left(\frac{\sigma_1(\sigma_2 \cdot \mathbf{q}_2)}{q_2^2 + m_\pi^2} - \frac{\sigma_2(\sigma_1 \cdot \mathbf{q}_1)}{q_1^2 + m_\pi^2} \right)$$

$$\hat{J}_V^{[2],\text{pif,nr}}(\mathbf{q}) = i \left(\frac{f_{\pi NN}}{m_\pi} \right)^2 (\vec{\tau}_1 \times \vec{\tau}_2)_\pm F_1^V(Q^2) \frac{(\sigma_1 \cdot \mathbf{q}_1)(\sigma_2 \cdot \mathbf{q}_2)}{(q_1^2 + m_\pi^2)(q_2^2 + m_\pi^2)} (\mathbf{q}_1 - \mathbf{q}_2)$$

with \pm corresponding with the incoming W^\pm

Ref: J. Ryckebusch, *et al.*, Nucl.Phys. A624, 581 (1997)

S. Janssen, *et al.*, Nucl.Phys. A672, 285 (2000)

(electron-scattering model with SRC + MEC)

Axial seagull currents ?

$$\hat{\rho}_A^{[2],\text{sea,nr}}(\mathbf{q}) = \frac{i}{g_A} \left(\frac{f_{\pi NN}}{m_\pi} \right)^2 (\vec{\tau}_1 \times \vec{\tau}_2)_\pm \left(F_\pi(Q_1^2) \frac{\sigma_2 \cdot \mathbf{q}_2}{q_2^2 + m_\pi^2} - F_\pi(Q_2^2) \frac{\sigma_1 \cdot \mathbf{q}_1}{q_1^2 + m_\pi^2} \right)$$

Summary and outlook

Summary SRC

- ▶ Started from a model for exclusive calculations which was tested against electron scattering data
- ▶ Calculated contribution of SRC to double differential QE cross section

Outlook

- ▶ Extending the model with meson-exchange currents in a consistent approach
 - ▶ Vector MEC model exists for electron scattering
 - ▶ Axial MEC are *challenging*

References

$2p2h$ e-scattering calculations including SRC and MEC

- ▶ J. Ryckebusch, *et al.*, Nucl.Phys. A568, 828 (1994)
- ▶ J. Ryckebusch, *et al.*, Nucl.Phys. A624, 581 (1997)
- ▶ S. Janssen, *et al.*, Nucl.Phys. A672, 285 (2000)

Momentum distributions with SRC

- ▶ J. Ryckebusch, *et al.*, J.Phys.G: Nucl.Part.Phys. 42 055104 (2015)

CRPA calculations for ν -interactions

- ▶ N. Jachowicz, *et al.*, Phys.Rev. C65, 025501 (2002)
- ▶ V. Pandey, *et al.*, Phys.Rev. C89, 024601 (2014)
- ▶ V. Pandey, *et al.*, Phys.Rev. C92, 024606 (2015)

Relativistic corrections

Relativistic kinematic corrections are implemented by the following simple substitution for ω (computed nonrelativistically) in the computation of the response functions W_i

$$W_i(\omega, q) \rightarrow W_i\left(\omega\left(1 + \frac{\omega}{2m_N}\right), q\right),$$

with m_N the nucleon mass. This can be interpreted as a shift of the QE peak from its nonrelativistic position to the relativistic position

$$\omega = \frac{q^2}{2m_N} \rightarrow \omega = \frac{Q^2}{2m_N}$$

- Ref: W. Alberico, *et al.* Nucl.Phys. A512, 541 (1990)
S. Jeschonnek, T.W. Donnelly, Phys.Rev. C57, 2438 (1998)
J. E. Amaro, *et al.*, Phys.Rev. C71, 065501 (2005)