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Extraction of N^* information from the limited $p(\gamma, K^+)\Lambda$ data set

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Abstract

Nucleon resonance information is often obtained from fitting hadrodynamical model calculations to data, where model parameters such as resonance coupling constants are the free parameters in the fitting procedure. For reactions with a limited data set, such as $p(\gamma, K^+)\Lambda$, complications in the extraction of reliable N^* information occur not only through theoretical uncertainties, but also due to technical difficulties in the fitting procedure. In this Letter we outline a fitting strategy based on a genetic algorithm and illustrate the kind of ambiguities which can arise.

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Understanding the composite, quark-gluon structure of hadrons is a major topic in intermediate energy nuclear physics. One way to investigate this system is to study its excitation spectrum, as it is a direct consequence of the hadron's composite character. At present, the experimental identification of the full excitation spectrum is far from complete, and most of our knowledge is based on analyses of pion induced or pion production reactions.

A complete understanding of the nucleon resonances can only be achieved by a full, coupled-channel analysis which incorporates reactions involving all possible initial and final states to which nucleon reso-

nances can couple [1–4]. Meson production reactions such as $\gamma N \rightarrow \pi N, \pi\pi N, \eta N, \omega N, K\Lambda, K\Sigma, \dots$, should be included, as well as Compton scattering and meson-induced production reactions. It is clear that a simultaneous description of all such reactions is an enormous task.

When the (photoinduced) meson production processes are treated at a hadronic level, the resonances are described by effective fields with independent quantities such as mass, strong decay widths and photo helicity amplitudes. Usually one determines these resonance parameters by fitting model calculations to the available data. The fitting procedures are therefore a necessary step in the extraction of resonance information from experimental measurements. However, by taking into account all known resonances in a full coupled-channel calculation, one would require a

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few *hundred* parameters to be determined. As a first step, a single-channel analysis of one particular reaction is essential to identify general features, and to simplify the problem by reducing it to one dependent on only a few *tens* of parameters.

The search for an optimum fit to the data, by a model depending on what is still a large number of parameters, is not straightforward. The situation is considerably worsened when reaction channels such as ωN , $K \Lambda$ or $K \Sigma$ are examined, since only limited data sets are available. In these cases, one may encounter the situation where model parameters are not sufficiently constrained by the restricted data sets and similar qualities-of-fit are obtained with different sets of parameter values. In this Letter, we highlight the issue by presenting a fitting procedure based on a genetic algorithm (GA) and commenting on some of the related ambiguities which arise in the complex fitting processes. We have chosen the $p(\gamma, K^+) \Lambda$ reaction as an example with a limited amount of published data, and where we can restrict the number of free parameters by employing a single-channel model.

In our model, the $p(\gamma, K^+) \Lambda$ reaction dynamics is described in terms of hadronic degrees of freedom by adopting an effective Lagrangian approach [5]. The tree level Feynman diagrams contain the usual Born terms, the $K^*(892)$ and $K_1(1270)$ t -channel mesons and two hyperon resonances $S_{01}(1800)$ and $P_{01}(1810)$ in the u -channel. All these terms constitute the so-called *background*. In the s -channel, the nucleon resonances $S_{11}(1650)$, $P_{11}(1710)$, $P_{13}(1720)$ and $D_{13}(1895)$ are included. Note that the $D_{13}(1895)$ is a candidate for a “missing” nucleon resonance [6]. The finite extension of the meson-baryon vertices is implemented by the use of hadronic form factors [7,8]. In Ref. [5] we stressed the difficulties associated with parameterizing the background diagrams in $p(\gamma, K^+) \Lambda$ calculations and presented results for three plausible background schemes. Subsequent work [9] showed that the $p(e, e' K^+) \Lambda$ process is highly selective with respect to viable choices for dealing with the background diagrams. The model used here is the only one which we found to reproduce simultaneously the $p(\gamma, K^+) \Lambda$ and $p(e, e' K^+) \Lambda$ data.

All the extracted resonance parameters $G[N^*]$ are a product of an electromagnetic and a strong coupling. A description of the various types of resonance para-

eters $G[N^*]$, their normalization and their connection to the Lagrangian structure is given in Ref. [5]. To minimize the number of free parameters, one overall hadronic form factor cutoff mass for all Born terms and one for all resonance diagrams are introduced. These coupling constants and cutoff masses constitute the free parameters of the model. In the present hadrodynamical model, this amounts to 22 free parameters which have to be determined by the data. We have used the SAPHIR data set [10] which contain 24 total cross section, 90 differential cross section and 12 recoil polarization asymmetry points, in the photon energy range from threshold up to 2 GeV. The fits are performed by minimizing χ^2 . Finding the global χ^2 minimum in a 22-dimensional parameter space is not trivial since although some of the parameters are constrained to a specific range by physics arguments, in principle they are all unknown quantities.

In order to minimize the χ^2 function and extract N^* information from the $p(\gamma, K^+) \Lambda$ data, we have used a *genetic algorithm*. In this strategy, a set of solutions is randomly generated. Each solution \mathbf{p} is an encoding of trial values of the free parameters, and is used to evaluate the $\chi^2(\mathbf{p})$ function which determines its “fitness”. The population is then “evolved” in a manner analogous to biological evolution, either by mixing values (“crossover”) or by changing values at random (“mutation”). The population gradually migrates to one or more better points in parameter space. Whilst these optima are not guaranteed to be global, GA research [11,12] has shown that “reasonable” solutions can be found very efficiently.

The output of the GA is the individual with the best fitness after a prescribed number of fitness function evaluations. The parameters associated with this individual are then used as input to the MINUIT [13] package which employs a variation of the Davidon–Fletcher–Powell variable-metric minimization algorithm. In this way we take advantage of the wide search capabilities of the GA, but use the strength of a traditional optimizer to obtain final solutions.

To test whether global optima were being found, 50 GA plus MINUIT calculations (run concurrently on a Linux-based PC farm) were carried out. Of these calculations, 38 resulted in MINUIT converging with a χ^2 below 3.00. This was thought to be a reasonable value, since our previous work [5] had shown χ^2 values of just under 3.00 to be achievable. The range of

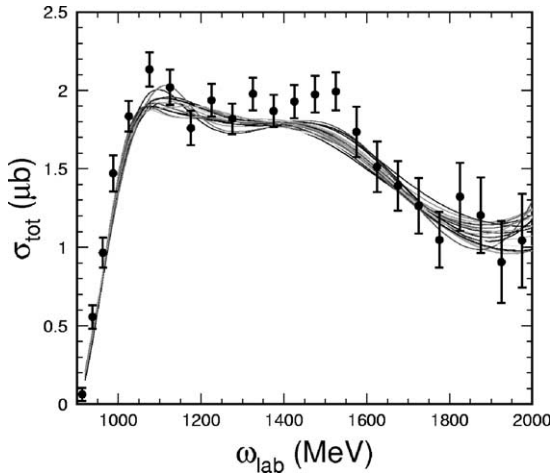


Fig. 1. Model calculations for the energy distribution of the total $p(\gamma, K^+)\Lambda$ cross section. The curves correspond to the 38 solutions with $\chi^2 \leq 3$. The data are from Ref. [10].

the fitness of these solutions¹ was $2.45 \leq \chi^2 \leq 2.93$, indicating a reasonable goodness-of-fit to the available data for *each* solution. This is apparent in Figs. 1, 2, and 3 where the model calculations for the total and differential cross section, and the recoil polarization asymmetry are plotted against SAPHIR data.

Given this similarity in the description of the available data, it is interesting to investigate how close these solutions of the minimization procedure are to each other in parameter space. In Fig. 4, resonance parameters for all the solutions are plotted. At a first glance, it appears that the resulting χ^2 surface in the free parameter space is extremely erratic and has a large number of local minima. The solutions for $G[P_{11}(1710)]$ show that the value could be anything within a wide range. On the other hand, the $S_{11}(1650)$ coupling constant exhibits clustering in one region, indicating that the value must be within a reasonably small range. Of great interest is the clustering exhibited in the $G^{(1)}[P_{13}(1720)]$ coupling constant. Here, there are three distinct clusters plus one isolated point. Furthermore, two clusters appear in the D_{13} parameters. We are therefore unable to pin down clear-cut values for all N^* parameters. This clearly illustrates the short-

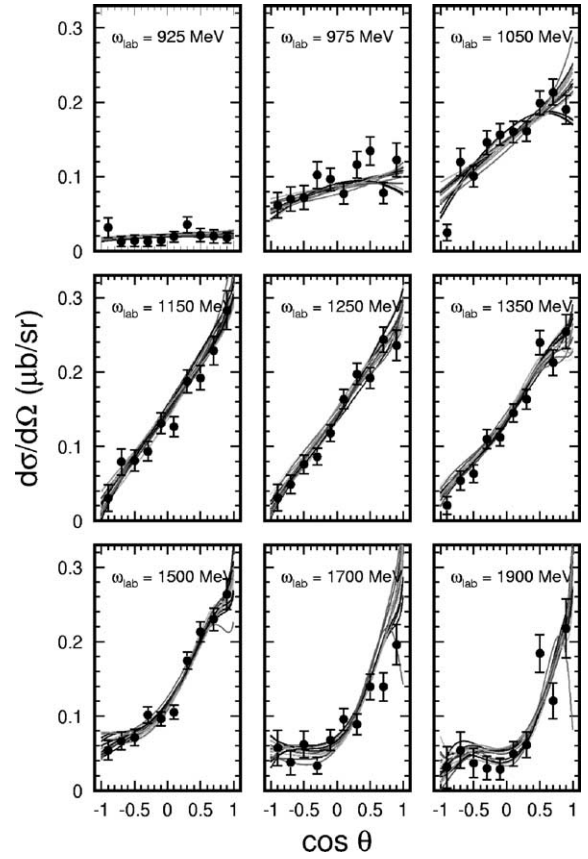


Fig. 2. Model calculations as in Fig. 1 but for the angular distribution of the differential $p(\gamma, K^+)\Lambda$ cross section. The data are from Ref. [10].

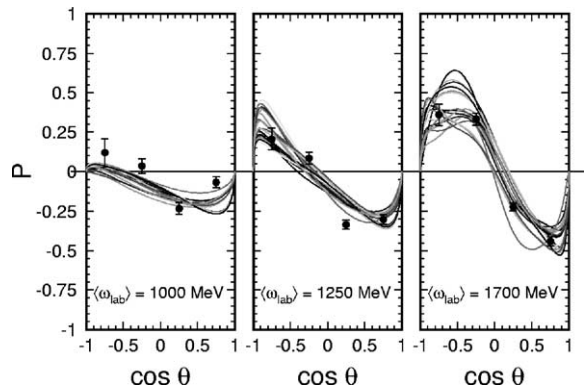


Fig. 3. Model calculations as in Fig. 1 but for the angular distribution of the $p(\gamma, K^+)\Lambda$ recoil polarization asymmetry. The data are from Ref. [10].

¹ From now on, we refer to “solutions” as meaning those in the converged set of 38.

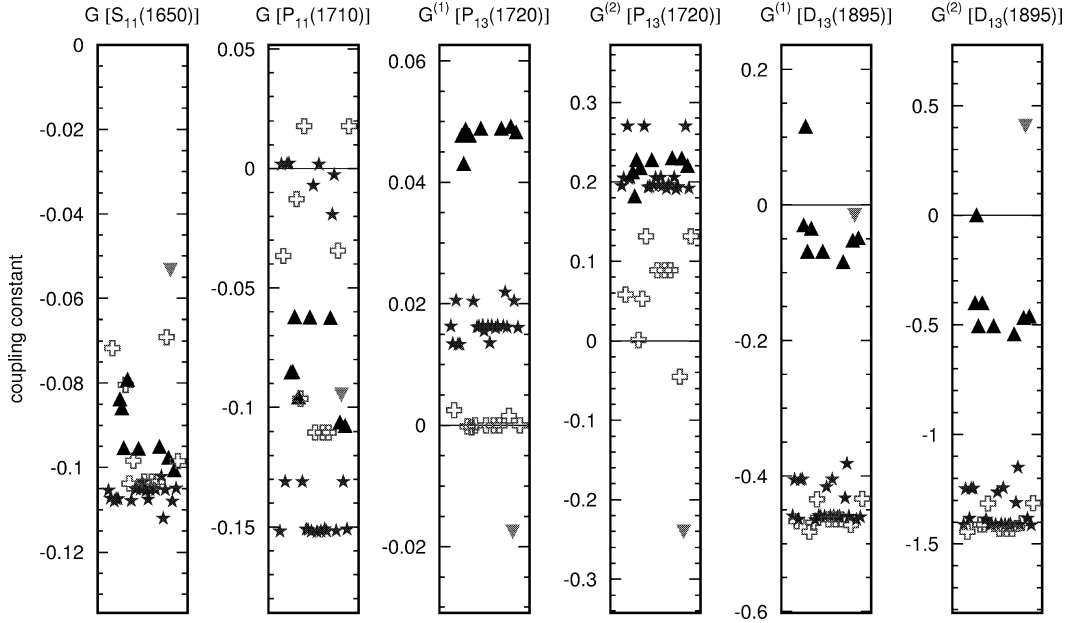


Fig. 4. Values of the coupling constants of the four N^* resonances $S_{11}(1650)$, $P_{11}(1710)$, $P_{13}(1720)$ and $D_{13}(1895)$ in the reaction dynamics of $p(\gamma, K^+)\Lambda$. They correspond to the best solutions of the minimization procedure. The four clusters are labeled by \blacktriangle , \blacktriangledown , \star and \oplus .

comings in extracting reliable resonance parameters from the available limited $p(\gamma, K^+)\Lambda$ data.

To study how the coupling constants are correlated, we have assigned each of the four clusters in the $G^{(1)}[P_{13}(1720)]$ parameter a distinct symbol. In Fig. 4, there appears to be a significant correlation between the P_{13} parameter and the D_{13} couplings. In the D_{13} , the \blacktriangle and \blacktriangledown clusters form one subset, whilst the \star and \oplus form another. Since the role of the $D_{13}(1895)$ in this $p(\gamma, K^+)\Lambda$ reaction is still under debate, we have split the solutions into two subsets according to this clustering in the D_{13} parameters. The characteristics of the two subsets are summarized in Table 1.

As a further study, we show model predictions for the two subsets for some unmeasured observables. In Figs. 5 and 6 this is done for the photon beam asymmetry Σ , the beam-recoil asymmetry O_x and the beam-target asymmetry E . Since none of these asymmetries for the $p(\gamma, K^+)\Lambda$ reaction are available yet, they do not represent constraints in the minimization procedure. Within each of the two subsets, the predicted kaon-angle dependence of Σ , O_x and E can be seen to follow similar trends (the dashed exception in Fig. 5 is the solitary \blacktriangledown solution).

Table 1

Characteristics of the two subsets localized in the solutions of the minimization procedure. The averaged $\langle\chi^2\rangle$, its standard deviation and the best χ^2 are given together with the number of points in the set and the corresponding labels in Fig 4

Subset	1	2
$\langle\chi^2\rangle$	2.56	2.69
σ_{χ^2}	0.13	0.12
Best χ^2	2.45	2.50
Number of points	9	29
Cluster labels	$\blacktriangle\blacktriangledown$	$\star\oplus$

More important discrepancies are observed when comparing the asymmetry distributions of the two subsets. The main difference appears to be a result of the value of the $D_{13}(1895)$ resonance parameters. It is interesting to note that the value of the $G[P_{11}(1710)]$ parameter appears to have only a small influence in the asymmetries. Reliable extraction of $P_{11}(1710)$ information from $p(\gamma, K^+)\Lambda$ data is therefore likely to be difficult, even when more (double) polarization asymmetries are available. The model predictions in Fig. 6 show that also the differences in the P_{13} parameters between the \star and \oplus solutions do not translate to significant differences in the plotted asymmetries. On the

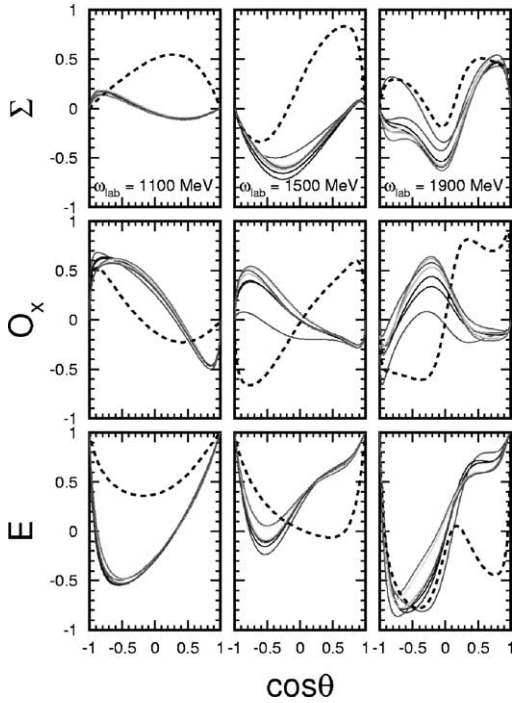


Fig. 5. Model predictions with the solutions of subset 1 (\blacktriangle and \blacktriangledown) for the angular distribution at various photon energies of the $p(\gamma, K^+)\Lambda$ photon beam asymmetry (Σ), the beam-recoil asymmetry (O_x) and the beam-target symmetry (E).

other hand, the calculation with the “odd” behavior in Fig. 5 is the solitary \blacktriangledown solution. Since this solution deviates from the \blacktriangle cluster most strongly in the P_{13} parameters, this shows a non-negligible influence of the $P_{13}(1720)$ resonance on the observables.

The calculations show that the present data cannot discriminate between two main regions in parameter space. The clear differences in the asymmetries between the two subsets will require only modest statistical accuracy to distinguish experimentally. Therefore, the upcoming data for the photon beam asymmetry [14,15] and the double polarized asymmetries [16] will be vital in constraining the values of resonance coupling constants and understanding the reaction mechanism underlying $K^+\Lambda$ production.

In conclusion, we have developed a fitting strategy based on a genetic algorithm in order to extract resonance information from $p(\gamma, K^+)\Lambda$ data. The advantages of the genetic algorithm are that the available parameter space is extensively sampled, and that the large number of calculations allows one to investigate

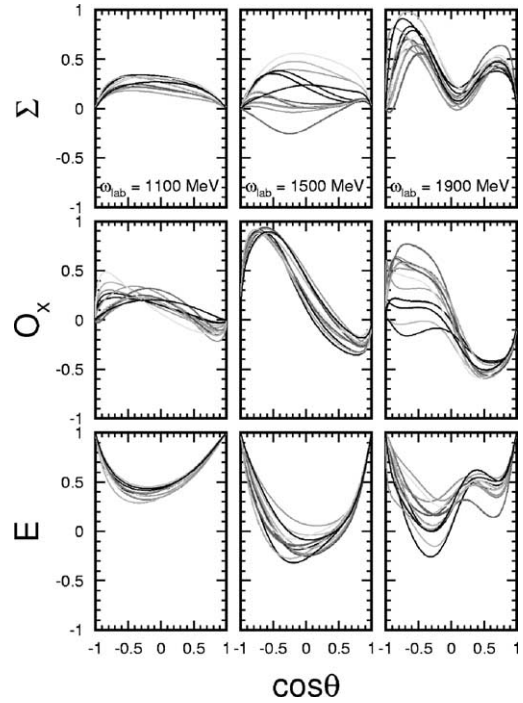


Fig. 6. Model calculations as in Fig. 5 but with the subset 2 (\star and \oplus).

any ambiguities in the solutions. The success of this strategy perhaps points to its use in problems with similar or larger numbers of free parameters.

The solutions obtained from the minimization runs result in model calculations in close agreement with the available data set. However, from a detailed exploration of the parameter values, it appears that not all solutions are contained in the same region of parameter space. For some of the variables, large deviations appear and no well defined value can be pinned down. This is particularly true for the $P_{11}(1710)$ coupling constant. For other N^* parameters, clustering in a specific area is observed. One stable cluster appears for the $S_{11}(1650)$ parameter and for the $P_{13}(1720)$ and $D_{13}(1895)$, a few isolated groups of solutions are identified. On the basis of these clusters in the N^* parameters, we were able to select two different subsets of solutions of the χ^2 minimization. Solutions within each subset produce more or less identical predictions for unmeasured polarization observables, but large differences were observed when comparing the two sets.

The calculations indicate that the extraction of reliable resonance information from a limited data set is by no means trivial, and has to be handled with the greatest of care. An important conclusion is that when using theoretical models to analyze the excitation spectrum of the nucleon based on a limited data set, the accuracy of predictions will be greatly affected by uncertainties stemming from the fitting process. Hence, employing a strategy which enables a thorough exploration of parameter space is essential. We have shown that the upcoming measurements of specific polarization asymmetries are essential to reduce the uncertainties resulting from the present limited data set.

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References

- [1] D. Manley, E. Saleski, *Phys. Rev. D* 45 (1992) 4002.
- [2] T. Vrana, S. Dytman, T.-S. Lee, *Phys. Rep.* 328 (1999) 181.
- [3] G. Penner, U. Mosel, *Phys. Rev. C* 66 (2002) 055211.
- [4] G. Penner, U. Mosel, *Phys. Rev. C* 66 (2002) 055212.
- [5] S. Janssen, J. Ryckebusch, D. Debruyne, T. Van Caueren, *Phys. Rev. C* 65 (2002) 015201.
- [6] T. Mart, C. Bennhold, *Phys. Rev. C* 61 (2000) 012201(R).
- [7] H. Haberzettl, *Phys. Rev. C* 56 (1997) 2041.
- [8] R. Davidson, R. Workman, *Phys. Rev. C* 63 (2001) 025210.
- [9] S. Janssen, J. Ryckebusch, T. Van Caueren, *nucl-th/0301096*.
- [10] M.Q. Tran, et al., *Phys. Lett. B* 445 (1998) 20.
- [11] D. Goldberg, *Genetic Algorithms in Search, Optimization and Machine Learning*, Addison–Wesley, Reading, MA, 1989.
- [12] L. Davis, *Handbook of Genetic Algorithms*, Van Nostrand–Reinhold, New York, 1991.
- [13] CERN, MINUIT 95.03, CERN library d506 Edition, 1995.
- [14] R.G.T. Zegers, et al., *nucl-ex/0302005*.
- [15] A. D’Angelo, in: D. Drechsel, L. Tiator (Eds.), *Proceedings of the NSTAR 2001 Workshop on The Physics of Excited Nucleons*, World Scientific, Singapore, 2001, p. 347.
- [16] F.J. Klein, et al., CLAS proposal e-02-112, TJNAF, Newport News, VA, USA, 2002.