RELATIVISTIC QUARK MODELS FOR BARYONS: THE EXTRAORDINARY WORLD OF ORDINARY MATTER.

Tamara Corthals

Ghent University, Department of Subatomic and Radiation Physics
Proeftuinstraat 86, 9000 Gent

We discuss the theoretical description of baryons in relativistic quark models. Special attention is paid to the model developed by the Bonn group. This model is remarkable in the sense that it manages to reconcile a field-theoretical framework with the phenomenological properties of baryons (e.g. confinement), and its formulation is fully Lorentz covariant. We have used this model to calculate baryon mass spectra, as well as electromagnetic properties of strange baryons.

1. The trouble with medium-energy

The words "fundamental theory of matter" have always had a certain ring to them. For centuries, man has striven to understand the world around him, to see order in places where at first sight there is none to be found. The search for a "theory of everything" might even be called one of the principal quests of contemporary science.

A landmark in our description of the microscopic world has undeniably been the development of the "standard model". This model classifies all elementary particles into two classes of fermions: leptons (e.g. the electron e and electron-neutrino ν_e) and quarks (which are the principal building blocks of protons and neutrons). It also brings order in three of the four fundamental forces through which the fermions interact: the weak, electromagnetic and strong interaction, which are mediated by the exchange of bosons, e.g. the photon in the electromagnetic case (see fig. 1).

For each of these three interactions, a field-theoretical framework is available. An understanding of the behaviour of the *fundamental* particles, however, doesn't necessarily guarantee an understanding of matter on a larger scale. In this respect the strong interaction, to which only the quarks are subject, is the most problematic one.

Figure 1: *The fundamental particles according to the standard model: six quarks (up, down, charm, strange, top, bottom, of which only u, d and (to a minor degree) s are found in "ordinary" (i.e. non-exotic) matter); six leptons (electron, muon and tau, plus the corresponding neutrinos); and four types of force-carrying bosons, for the electromagnetic (γ), strong (g) and weak (Z, W) interaction. Each particle has an antiparticle "twin" as well.* (image source: http://www.fnal.gov)

It turns out to be impossible to find a complete theoretical description of systems composed of several quarks, starting from the fundamental equations of quantum chromodynamics (QCD), the field theory of strong interactions. The reason for this is, that the strong quark-quark interaction becomes *stronger* at larger distances (or, lower energies). A fascinating and theoretically challenging side effect of this property is the fact that quarks can *never* be observed as free objects, but only as bound states of several quarks and/or antiquarks, called hadrons[1]. This phenomenon, called "confinement", makes it impossible to gather experimental information about quarks in a *direct* manner. Moreover, the QCD equations can only be solved through the use of perturbation theory at sufficiently high energies. Thus, perturbative QCD is adequate for high-energy processes,

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[1] Hadrons are commonly classified into three-quark systems or baryons, and quark-antiquark systems or mesons. Recent experiments provide strong indications for the existence of more exotic hadrons (e.g. the recently discovered pentaquark [1]).

but in the energy region relevant to the study of "ordinary" matter, the equations simply cannot be solved exactly[2]. Even though the development of high-energy particle accelerators has caused an upsurge of interest in QCD at small length scales, the study of "ordinary" matter ought not to be forgotten. The title of this article suggests that exciting physical phenomena occur in nucleons and nuclei as they are found under earthly circumstances.

In hadronic physics, which is the domain aimed at unravelling the structure of hadrons, models are an invaluable tool for linking observables to the fundamental theory. In the next two sections, we will shed some light onto the typical experiments in hadronic physics, and the theoretical tools necessary to interpret them.

2. Electromagnetic screwdrivers

An experimental investigation of quarks can only be conducted indirectly. Thereby, hadron properties and hadron spectroscopy are vital sources of information. One of the principal objectives of ongoing experimental efforts is to improve our knowledge about the nucleon spectrum. Just like the excited states of atoms teach important lessons about e.g. the electron-nucleus interaction, knowledge of the possible nucleon excitations, or "resonances" (commonly abbreviated as N^*), provides the clues which help to understand the behaviour of quarks at nuclear and hadronic scales.

A lot of attention is paid to the subject of the "missing resonances", which are predicted by quark models but have not (yet) been observed experimentally. A typical reaction in which to search for new nucleon resonances, is pion-nucleon (πN) scattering. In this process, a pion and a nucleon can momentarily form an excited nucleon state, which instantly decays into the initial particles (see fig. 2). Albeit powerful, $\pi N \to \pi N$ reactions do not allow us to acquire a complete picture of the N^* spectrum. Indeed, some nucleon resonances are expected to couple only very weakly to the pion-nucleon channel (i.e. they rarely decay into a π and N), which would make them almost impossible to

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[2] An extrapolating, numerical approach to QCD in the medium-energy region, called "lattice QCD", does exist, but up to now only static hadron properties have been computed in this manner.

observe. However, there is reason to believe that some of these resonances will have a larger probability of decaying into hadrons containing *strange* (= s) quarks - specifically, a baryon containing an s-quark (a so-called "hyperon"), and a meson containing an s-antiquark. For this reason, "open strangeness production" experiments, i.e. scattering experiments in which hyperons (Y) and kaons (K) are produced, are presently being conducted at several experimental facilities [2,3,4].

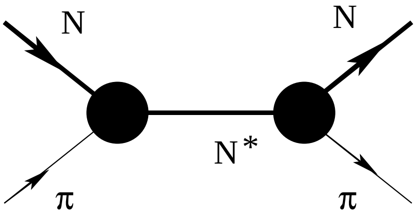

Figure 2: *A possible process contributing to the elastic πN scattering cross-section: formation of an intermediate-state nucleon resonance.*

In all these experimental investigations, an important question is, which particles to scatter from a nucleon? As the electromagnetic interaction is well understood, one logical choice is the scattering of real photons off nucleons. Alternatively, electron scattering can be used, since electrons interact with hadrons through the exchange of a virtual photon. Since the electromagnetic interaction is much weaker than the strong force binding the quarks into a nucleon, the disturbance of the hadron under investigation by the photon's presence will be minimal. In that sense one uses the photon as a tiny "screwdriver" in order to probe the nucleon's internal structure.

3. A theorist's toolbox for strangeness production

For the theoretical description of strangeness production processes, there are several viable options, depending amongst other things on the energy region in which one is interested. Our group has developed models based on Quantum Hadro-Dynamics (QHD), which is a field theory with hadrons as effective degrees of freedom [5]. In the QHD framework, the quark structure of the hadrons is not explicitly taken into account, but is parameterized. In practice, an effective field theory is formally completely analogous to the fundamental field theories, except that the precise mathematical structure, as well as the strengths of the effective hadronic interactions, are not *a priori* known. They either need to be determined experimentally, or computed in a more fundamental model. Because of practical difficulties, not all quantities necessary for an unambiguous theoretical QHD description of the process can be experimentally determined. Therefore, quark models are an invaluable tool for obtaining additional information. This was precisely the subject of my Master's thesis: using quark models to determine some of these hard-to-measure quantities. The ultimate goal is obtaining an improved description of strangeness-production processes. These, in turn, will help us piece together the puzzle of the nucleon spectrum and hadron structure in general.

We focus on the specific reaction shown in fig. 3, namely the scattering of a real or virtual photon from a nucleon, with production of a kaon (K) and a strangeness[3] $S = -1$ hyperon (Y). For this process, several scenarios are possible, all of which should in principle be included in the calculations. Figure 3 shows Feynman diagrams for two examples where the intermediate state is a nucleon (N^*, fig. 3a) and a hyperon (Λ^* or Σ^*, fig. 3b) resonance.

In quantum field theory, each particle line and interaction vertex appearing in a Feynman diagram is replaced by a corresponding mathematical structure when computing the transition matrix elements. The unknown mathematical quantities are: the structure of the vertex factors, the coupling strengths corresponding to these vertex factors, and the particle propagators (depending on the mass, spin and decay width). As alluded to above, one of the major differences between an effective and a fundamental field theory is, that fundamental particles are point-like, while effective particles (in this case, hadrons) can possess a finite size. This finite size can be modelled by allowing the coupling strength at the electromagnetic vertex to become dependent on the energy, or more

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[3] Strangeness S is an additive quantum number indicating the number of strange quarks, where $S(s) = -1$ and $S(\bar{s}) = +1$. A kaon is a meson containing one $\bar{s}$ quark, along with either u or d. There are two classes of $S = -1$ hyperons: the Λ (uds), and three Σ particles (uus, uds, dds).

specifically, on the four-momentum squared (Q^2) of the exchanged virtual photon.[4] These Q^2-dependent coupling strengths are called "electromagnetic form factors". For the same reasons, *hadronic* form factors will appear at the strong interaction vertices. Here, we focus on the electromagnetic ones.

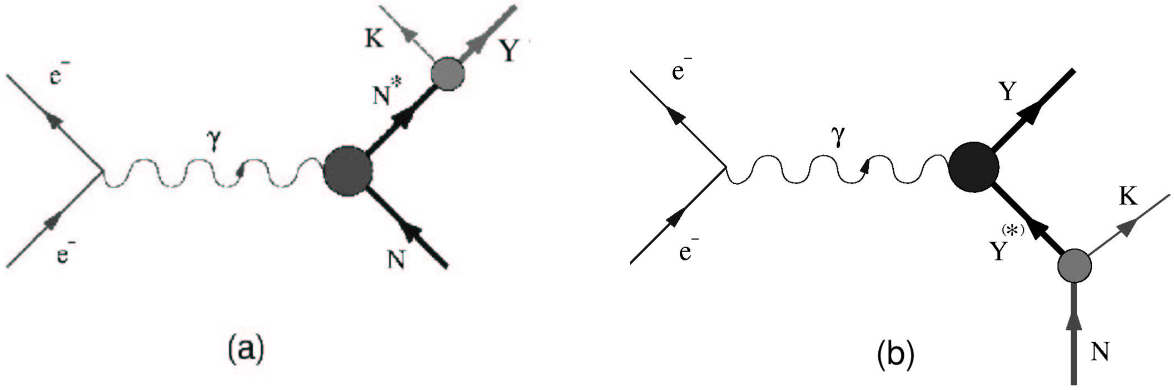

Figure 3: *Two examples of intermediate states occurring in an $e + N \to e' + K + Y$ process. In figure (a), the nucleon couples to the virtual photon, and a nucleon resonance is formed, which subsequently decays into a kaon and a hyperon. In figure (b), the nucleon emits a kaon and turns into a hyperon resonance, which then couples to the photon to form a ground-state hyperon.*

The electromagnetic form factors for the proton and neutron are reasonably well known. For most other baryons, they are more difficult to measure. This is especially the case for the hyperon form factors, because of the instability of these particles. Thus, theoretical predictions of form factors in a quark model are essential in attempting to unravel the dynamics of strangeness production processes. In the next section, we will sketch the features and assets of the relativistic quark model we adopted to compute the form factors, and present some results of our calculations.

4. To capture relativity

The popularity of non-relativistic models in the hadronic physics community is as impressive as it is surprising. Despite the fact that quarks swirl in the interior of hadrons at

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[4] Through the Heisenberg relation $\Delta p \Delta x \approx \hbar$, this energy-dependence corresponds to a dependence of the interaction strength on the distance, or the resolution with which the photon "sees" the hadrons.

relativistic speeds, non-relativistic models do a remarkable job in describing the hadron spectrum. This is partly due to the quite large number of parameters introduced in these models. Furthermore, in non-relativistic models the spurious center-of-mass degrees of freedom can easily be eliminated. However, when it comes to form factors, especially at larger Q^2 values, strong relativistic effects are expected to emerge.

The quark model we use was developed by the Bonn group [6,7,8] and it has the fundamental property of being fully Lorentz covariant. This allows for a correct formulation of Lorentz boosts, which is a vital condition in the treatment of higher-energy transitions. It is essentially a field-theoretical framework, in which the unknown quantities are modelled in a phenomenological way. The main difference between the Bonn-model field theory and QCD lies in the fact that the Bonn model does not consider the fundamental (or "current") quarks as degrees of freedom, but the so-called "constituent" quarks (of which a baryon contains three, and a meson two). These are essentially current quarks surrounded by a cloud of gluons and quark-antiquark pairs, which modifies their properties. So the Bonn model is again an effective theory, but at a deeper level than the purely hadronic theories.

Still, the Bonn model suffers from the same "curse" as most effective theories do, namely some of the necessary model ingredients are not *a priori* known [6]. The precise form of the two- and three-quark interactions, for example, can only be determined by "trial and error", and through the inclusion of some necessary phenomenology. For example, the quark-quark confinement interaction linearly rises with the inter-quark distance, as hinted at by experiments and lattice calculations. For the two-quark residual interaction, a QCD-based potential suggested by 't Hooft is adopted. These and some other approximations eventually lead to a mathematically manageable (though involving) framework, which is very well suited for the description of static hadron properties like masses and electromagnetic form factors. We have focused on the latter, specifically for hyperons. Still, it deserves to be mentioned that the Bonn model gives an excellent description of the hadron spectrum [6].

We present some results of our form-factor calculations below. In figure 4, the elastic electric and magnetic form factors G_E and G_M of the strangeness $S = -1$ ground-state hyperons are displayed. We compare our predictions with an alternative model calculation by Kim *et al.* [9].

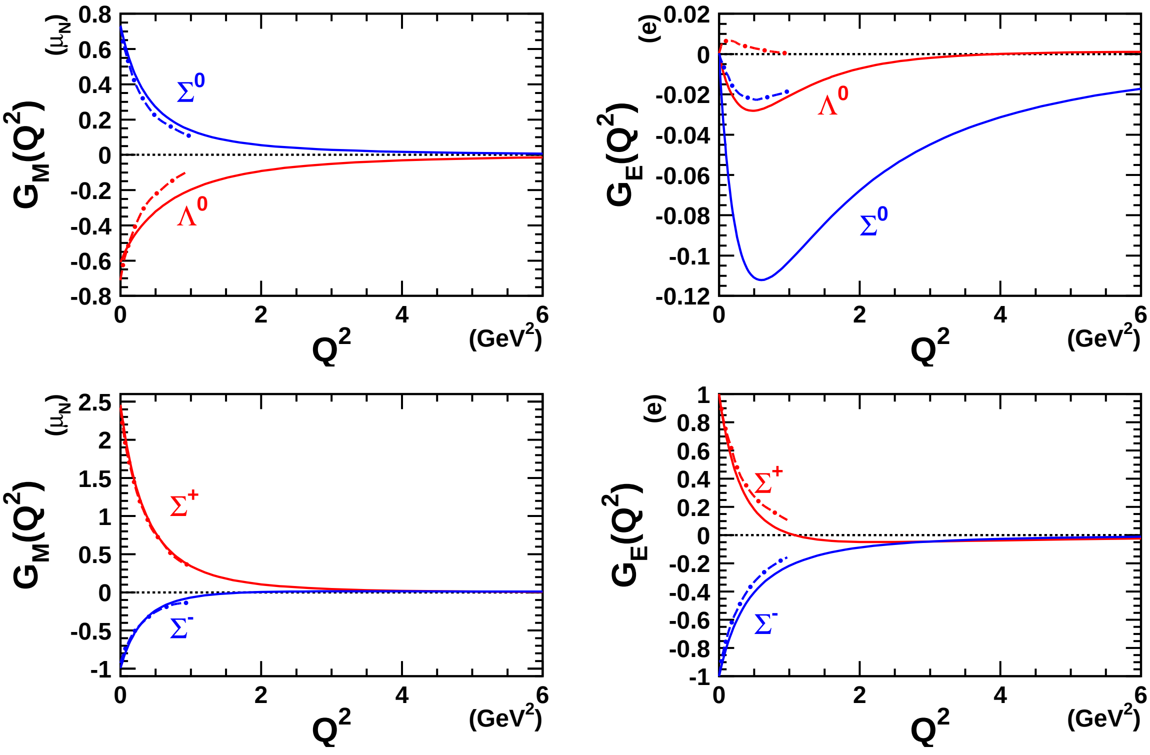

Figure 4: *The electric and magnetic form factors* $G_{E,M}(Q^2)$ *for the neutral* Σ^0 *and* Λ^0 *hyperons (top), and for the charged* $\Sigma^\pm$ *hyperons (bottom). The dot-dashed curves are the predictions from [9].*

Table 1 summarizes the magnetic moments for the ground-state hyperons, i.e. the values of the magnetic form factors at $Q^2 = 0$. The magnetic moment is one of the rare hyperon properties for which a comparison to experiment is possible at this time.

Finally, in figure 5, we show the Dirac and Pauli magnetic form factors F_1 and F_2 for the electromagnetic $\Sigma^0 \rightarrow \Lambda$ transition. Additional results, as well as a more detailed discussion of the figures and table displayed here, are presented in [13].

Baryon	μ_Y^{exp}	μ_Y^{calc}	$\mu_Y^{[9]}$	$\mu^{[10]}_{Y\,(GBE/OGE)}$	$\mu^{[11]}_{Y\,(HB/IR)}$
$\Lambda^0(1116)$	-0.613 ± 0.004	-0.61	-0.77	$-0.59/-0.59$	exp.
$\Sigma^+(1189)$	2.458 ± 0.010	2.47	2.42	2.34/2.20	exp.
$\Sigma^0(1189)$	0.649	0.73	0.75	0.70/0.66	exp.
$\Sigma^-(1189)$	-1.160 ± 0.025	-0.99	-0.92	$-0.94/-0.89$	exp.
$\lvert\Sigma^0 \to \Lambda\rvert$	1.61 ± 0.08	1.41	1.51	—	1.46/1.61
$\Xi^0(1315)$	-1.250 ± 0.014	-1.33	-1.64	$-1.27/-1.27$	exp.
$\Xi^-(1315)$	-0.6507 ± 0.0025	-0.57	-0.68	$-0.67/-0.57$	exp.

Table 1: *Magnetic moments of strange baryons in units of μ_N. The notation* GBE/OGE *(*HB,IR*) refers to the two different models discussed in [10] ([11]). In [11], only the transition magnetic moment for $\Sigma^0 \to \Lambda$ is a real prediction. Experimental values are taken from [12], except for $\mu_{\Sigma^0} = (\mu_{\Sigma^+} + \mu_{\Sigma^-})/2$, for which isospin invariance is used. For the $\Sigma^0 \to \Lambda$ transition, the absolute value is given.*

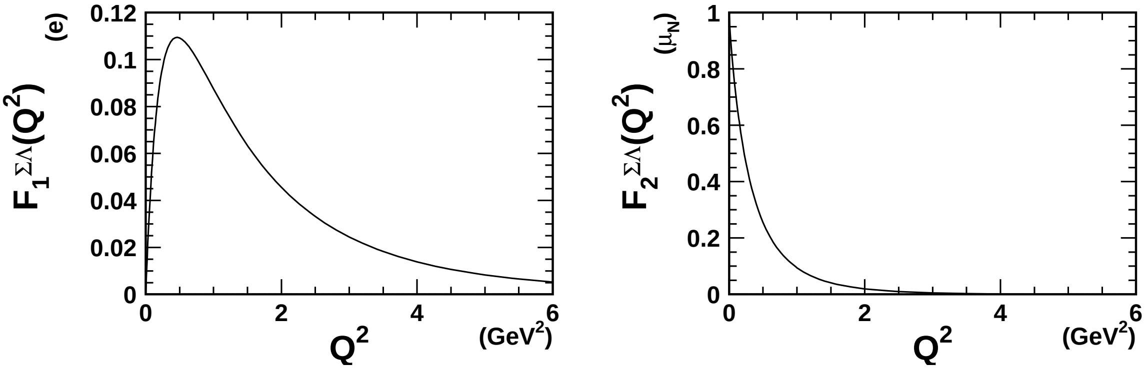

Figure 5: *The transition form factors of the $\gamma^* + \Sigma^0 \to \Lambda$ decay.*

5. Conclusion: hyperon form factors - where model beats experiment

In summary, the success of the Bonn quark model in describing static hadron properties can hardly be denied. First, it gives an impressive description of baryon mass spectra. Moreover, for those non-strange form factors which have already been measured, the agreement between the Bonn-model predictions and the experimental data is found to be quite satisfying [7]. Therefore, we believe we can feel rather confident in the reliability of our hyperon form-factor results, despite the fact that they cannot directly be compared to the experiment. The conclusion appears to be that quark models in general, and relativistic models in particular, allow us to circumvent the “dead end” at which the intractability of QCD at the nucleon’s distance scale had left us.

Presently, we are working on the implementation of these electromagnetic form factors in our hadrodynamic model for strangeness electroproduction. We are hopeful that this will provide us with new insights into the extraordinary phenomena governing the energy region of matter under earthly conditions.

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