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Hunting the Resonances in $p(\gamma, K^+)\Lambda$: (Over)Complete Measurements and Partial-Wave Analyses

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At given kinematics and for a given reaction, a complete set is a minimum set of observables from which one can unambiguously determine the underlying reaction amplitudes. For pseudoscalar-meson photoproduction, complete sets of eight observables which hold great promise are the combination of the differential cross section, of the three single-polarization observables and a proper selection of four double-polarization observables. Amplitude extraction is a model-independent way of extracting information from the data and is often being put forward as an alternate method for model falsification and for testing hypotheses about the underlying reaction dynamics. Information about the s-channel resonances is commonly extracted from the data by performing a partial-wave analysis. We apply both analysis techniques to the background-dominated $p(\gamma, K^+)\Lambda$ reaction. We introduce a variable that quantifies the maximum resolution that can be obtained in amplitude space after measuring a large amount of polarization observables.

KEYWORDS: Pseudoscalar-meson photoproduction, complete experiment, transversity basis, nucleon resonance extraction, polarization observables

1. Introduction

The four reaction amplitudes $\mathcal{M}_{i=1,2,3,4}$ for pseudoscalar-meson photoproduction depend on the invariant mass W and the scattering angle $\theta_{\text{c.m.}}$, two quantities that fully determine the reaction kinematics. The reaction amplitudes are connected to 16 observables at given kinematics (W, $\cos\theta_{\text{c.m.}}$). These observables consist of an unpolarized cross-section ($\frac{\mathrm{d}\sigma}{\mathrm{d}\Omega}$), 3 single-polarization observables (Σ, T, P) and 12 double-polarization observables (C_x, O_x, C_z, O_z ; E, F, G, H ; T_x, L_x, T_z, L_z). One can categorize the latter in three groups: the beam-recoil ($\mathcal{BR}$), beam-target ($\mathcal{BT}$) and target-recoil ($\mathcal{TR}$) observables respectively. With complete experiments one aims at determining the $\mathcal{M}_{i=1,2,3,4}$ over a range in (W, $\cos\theta_{\text{c.m.}}$). Since complete experiments involve single- and double-polarization measurements, the self-analyzing property of the Λ makes that the $p(\gamma, K^+)\Lambda$ reaction channel is likely to become the first channel for which it can be experimentally realized.

The reaction amplitudes $\mathcal{M}_{i=1,2,3,4}$ are defined for a given photon beam ($\mathcal{B}$), recoil hadron ($\mathcal{R}$) and target nucleon ($\mathcal{T}$) spin-quantization axis. We define the z-axis along the three-momentum of the impinging photon and the xz-plane as the reaction plane. The direction of the x-axis is such that the x-component of the meson momentum is positive. It has been pointed out [1] that with the eye on amplitude extraction the so-called transversity basis offers great advantages over all other common choices for the quantization axes. It is defined as

$$b_1 = \boldsymbol{T}^{y}_{+y,+y}\,, \qquad b_2 = \boldsymbol{T}^{y}_{-y,-y}\,, \qquad b_3 = \boldsymbol{T}^{x}_{-y,+y}\,, \qquad b_4 = \boldsymbol{T}^{x}_{+y,-y}\,, \tag{1}$$

where $\boldsymbol{T}^{\mathcal{B}}_{\mathcal{T},\mathcal{R}}$ is the reaction matrix-element for a given $\mathcal{B}$, $\mathcal{T}$ and $\mathcal{R}$ polarization. The $\mathcal{R} = \pm y$ ($\mathcal{T} = \pm y$) denotes a recoil (target) spin quantum number $\pm\frac{1}{2}$ along the y direction. The $\mathcal{B} = x, y$ denotes a lin-

early polarized photon along the x, y axis.

Another commonly used basis to define the reaction amplitudes is the CGLN representation [2], defined as the complex coefficients ($F_{i=1,2,3,4}$) of four independent operators. The F_{CGLN} spin-momentum operator adopts the form

$$F_{\mathrm{CGLN}} \equiv -iF_1\boldsymbol{\sigma}.\mathbf{e}_{\mathbf{P}^\gamma} - F_2\left(\boldsymbol{\sigma}.\mathbf{e}_{\mathbf{p}}\right)\left[\boldsymbol{\sigma}.\left(\mathbf{e}_{\mathbf{k}}\times\mathbf{e}_{\mathbf{P}^\gamma}\right)\right] - iF_3\left(\boldsymbol{\sigma}.\mathbf{e}_{\mathbf{k}}\right)\left(\mathbf{e}_{\mathbf{p}}.\mathbf{e}_{\mathbf{P}^\gamma}\right) - iF_4\left(\boldsymbol{\sigma}.\mathbf{e}_{\mathbf{p}}\right)\left(\mathbf{e}_{\mathbf{p}}.\mathbf{e}_{\mathbf{P}^\gamma}\right). \quad (2)$$

Here, the $\boldsymbol{\sigma} = (\sigma_x, \sigma_y, \sigma_z)$ are the Pauli matrices and $\mathbf{e}_{\mathbf{v}} = \mathbf{v}/|\mathbf{v}|$. The vectors $\mathbf{P}^\gamma$, $\mathbf{p}$, $\mathbf{k}$ denote the photon polarization, and the kaon and proton centre-of-mass (c.m.) momenta.

We denote differential cross section in c.m. frame, for a specific $\mathcal{B}$, $\mathcal{T}$ and $\mathcal{R}$ polarization as

$$\frac{\mathrm{d}\sigma^{(\mathcal{B},\mathcal{T},\mathcal{R})}}{\mathrm{d}\Omega}. \quad (3)$$

A $\mathcal{B} = 0$ ($\mathcal{T} = 0$) implies averaging over the two beam (target) polarizations, whereas $\mathcal{R} = 0$ implies a summation over the two recoil polarizations. The F_{CGLN} in Eq. (2) is directly related to the cross section

$$\frac{\mathrm{d}\sigma^{(\mathcal{B},\mathcal{T},\mathcal{R})}}{\mathrm{d}\Omega} \equiv \frac{|\mathbf{p}|}{|\mathbf{k}|}\left|\langle m_{s_\Lambda}(\mathcal{R})|F_{\mathrm{CGLN}}(\mathcal{B})|m_{s_p}(\mathcal{T})\rangle\right|^2, \quad (4)$$

where $|m_{s_\Lambda}\rangle$ ($|m_{s_p}\rangle$) is the usual eigenstate of the Pauli σ_z matrix. The transversity and CGLN amplitudes are connected through a linear transformation, which depends on $\theta_{\mathrm{c.m.}}$.

2. Multipole extraction

One commonly expands the CGLN amplitudes in a multipole decomposition [3]

$$F_1 = \sum_{l=0} P'_{l+1}(\cos\theta_{\mathrm{c.m.}})\left[E_{l+} + lM_{l+}\right] + P'_{l-1}(\cos\theta_{\mathrm{c.m.}})\left[E_{l-} + (l+1)M_{l-}\right] \quad (5)$$

$$F_2 = \sum_{l=1} P'_{l}(\cos\theta_{\mathrm{c.m.}})\left[(l+1)M_{l+} + lM_{l-}\right] \quad (6)$$

$$F_3 = \sum_{l=1} P''_{l+1}(\cos\theta_{\mathrm{c.m.}})\left[E_{l+} - M_{l+}\right] + P''_{l-1}(\cos\theta_{\mathrm{c.m.}})\left[E_{l-} + M_{l-}\right] \quad (7)$$

$$F_4 = \sum_{l=2} P''_{l}(\cos\theta_{\mathrm{c.m.}})\left[-E_{l-} - M_{l-} - E_{l+} + M_{l+}\right] \quad (8)$$

where the complex multipoles $E_{l\pm}$ ($M_{l\pm}$) quantify the contribution of spin $J = l \pm 1$ transitions in the s-channel induced by the electric (magnetic) component of the photon. The l denotes the angular momentum in the final state and P'_l (P''_l) denotes the l^{th} order Legendre polynomial. In practice, the infinite sum in Eqs. (5)–(8) is truncated at some L_{max}, a procedure that introduces some model dependence.

The RPR-2011 model [4–6] has been proposed as a single-channel reaction model for the $p(\gamma, K^+)\Lambda$ process. This model consists of a Reggeized t-channel background enriched with a number of nucleon resonances, determined through a Bayesian analysis of the available experimental data. The resonances that were found to substantially contribute are $S_{11}(1535)$, $S_{11}(1650)$, $F_{15}(1680)$, $P_{13}(1720)$, $D_{13}(1900)$, $P_{13}(1900)$, $P_{11}(1900)$, and $F_{15}(2000)$. A convenient feature of the RPR-2011 model is that there is clear separation between the background and resonant contributions. This allows one to investigate the effect of the background on the multipoles. The RPR-2011 multipoles for $p(\gamma, K^+)\Lambda$ are

displayed in Fig. 1. Also included are the separated Regge background and resonance contributions. We find that the multipoles are dominated by the background. Hence, the extraction of resonance information from multipoles in $p(\gamma, K^+)\Lambda$ hinges on the availability of reliable background-subtraction models.

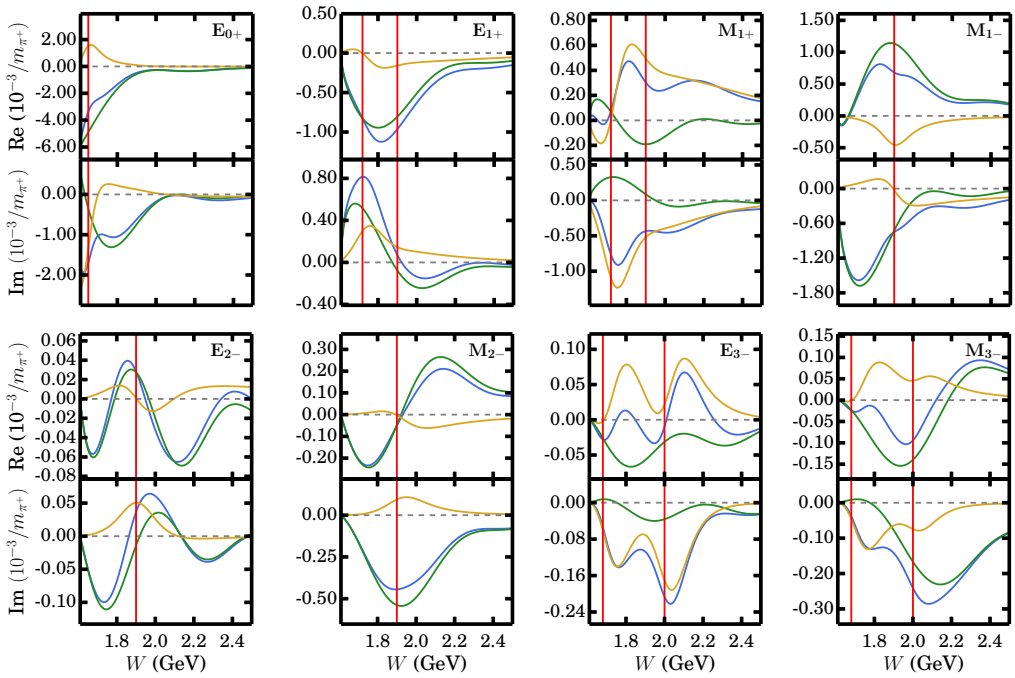

Fig. 1. The energy dependence of the $p(\gamma, K^+)\Lambda$ multipoles of the RPR-2011 model (blue). Also shown are the RPR-2011 background multipoles (green) and the RPR-2011 resonance multipoles (yellow). The vertical red lines indicate the position of the invariant mass of the contributing resonances.

3. Amplitude extraction

An asymmetry A is defined as

$$A\left(\mathcal{B}_1,\mathcal{T}_1,\mathcal{R}_1;\mathcal{B}_2,\mathcal{T}_2,\mathcal{R}_2\right)=\frac{\frac{\mathrm{d}\sigma}{\mathrm{d}\Omega}^{(\mathcal{B}_1,\mathcal{T}_1,\mathcal{R}_1)}-\frac{\mathrm{d}\sigma}{\mathrm{d}\Omega}^{(\mathcal{B}_2,\mathcal{T}_2,\mathcal{R}_2)}}{\frac{\mathrm{d}\sigma}{\mathrm{d}\Omega}^{(\mathcal{B}_1,\mathcal{T}_1,\mathcal{R}_1)}+\frac{\mathrm{d}\sigma}{\mathrm{d}\Omega}^{(\mathcal{B}_2,\mathcal{T}_2,\mathcal{R}_2)}}\,. \tag{9}$$

A typical example of a single-polarization and of a double-polarization asymmetry is $A\left(\mathcal{B}_1,0,0;\mathcal{B}_2\neq\mathcal{B}_1,0,0\right)$ and $A\left(\mathcal{B}_1\neq 0,\mathcal{T}_1,0;\mathcal{B}_2=\mathcal{B}_1,\mathcal{T}_2\neq\mathcal{T}_1,0\right)$. The observables are related to the complex reaction amplitudes by bilinear relations [1]. Since the expressions for the asymmetries are invariant under the scaling operation $b_i\rightarrow b_i/\sqrt{Q=\sum_{i=1}^{i=4}|b_i|^2}$, we introduce the normalized transversity amplitudes (NTA)

$$a_i(W,\cos\theta_{\mathrm{c.m.}})\equiv\frac{b_i(W,\cos\theta_{\mathrm{c.m.}})}{\sqrt{Q(W,\cos\theta_{\mathrm{c.m.}})}}\,. \tag{10}$$

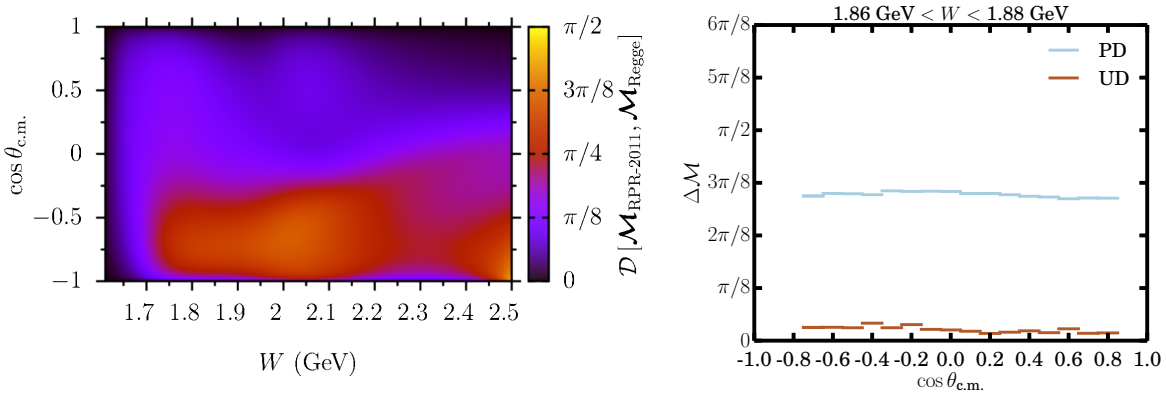

Fig. 2. The left panel shows the kinematic dependence of the distance measure of Eq. (12) between the full (resonance + background) RPR-2011 predictions for $p(\gamma, K^+)\Lambda$ and the RPR-2011 predictions including only the background. The right panel shows the angular dependence of the resolution of the published data (PD) and upcoming $\overset{(\rightarrow)}{p}(\overset{(\rightarrow)}{\gamma}, K^+)\overset{(\rightarrow)}{\Lambda}$ data (UD) in the bin $1.86 \leq W \leq 1.88$ GeV.

The expressions that connect the NTA to the asymmetries are summarized in Ref. [1]. In an amplitude analysis, one extracts the transversity amplitudes from a given set of observables in a model-independent way. Chiang and Tabakin showed that eight well-chosen observables constitute a complete set [7]. Hence, after parametrizing the NTA as $a_i \equiv r_i e^{\alpha_i}$ and after introducing the relative phases $\delta_i^{\alpha_4} = \alpha_i - \alpha_4$ (α_4 is defined as the global phase that cannot be determined experimentally) a complete set is the minimum amount of observables which is necessary to infer $(r_{i=1,2,3,4}, \delta_{j=1,2,3}, Q)$ unambiguously at given $(W, \cos\theta_{\text{c.m.}})$.

It has been demonstrated in Refs. [1, 8] that using single-polarization data one can already map the angular and energy dependence of the r_i for $p(\gamma, K^+)\Lambda$ to high accuracy over a wide range of kinematics (1.65 GeV $\leq W \leq 1.91$ GeV and $-0.81 \leq \cos\theta_{\text{c.m.}} \leq 0.86$). The obtained agreement between the RPR-2011 predictions for the $r_i(W, \cos\theta_{\text{c.m.}})$ and those extracted from the single-polarization $p(\gamma, K^+)\Lambda$ data, can be interpreted as a clear experimental verification for the t-channel dominance of the reaction.

The extraction of the δ_i is more challenging as it requires data for at least four double-polarization observables [9]. We have shown in Refs. [1, 8] that a mathematically complete set does not allow one to unambiguously extract the phases of the NTA. Indeed, for data with finite error bars, continuous and discrete ambiguities hamper the extraction of the phases. To resolve these ambiguities, measurements of overcomplete sets are necessary. Sets that include a complete set plus some additional polarization observables are referred to as overcomplete sets.

Amplitude extraction offers great opportunities for model comparisons and the question is what level of detail is required before one can let the data decide which model is preferred. To this end, we introduce a distance measure in amplitude space. We choose to clearly differentiate between information that follows from unpolarized cross section measurements and information from polarization measurements. In the following, we elaborate on the latter. We will map the distance between two models in amplitude space as a function of $(W, \cos\theta_{\text{c.m.}})$. This will be compared to the resolution of model-independent amplitude extractions on experimental data of polarization measurements.

To calculate the distance between two models in amplitude space, we need a distance measure

that includes all constraints. First, we introduce the $4D$-vector representation of the NTA

$$\boldsymbol{\mathcal{M}}_a = (a_1 \;\; a_2 \;\; a_3 \;\; a_4)^T \;\; , \tag{11}$$

which obeys the normalization condition $\boldsymbol{\mathcal{M}}_a^\dagger \boldsymbol{\mathcal{M}}_a = \sum_{i=1}^{4} |a_i|^2 = 1$. The 3-sphere in the $\mathbb{C}^4$ representation (11) can be mapped onto a geometrically equivalent 7-sphere in $\mathbb{R}^8$. This analogy provides one with an expression for a distance in $\mathbb{C}^4$, measured along the sphere defined by the normalization condition. For two amplitude sets $\boldsymbol{\mathcal{M}}_1$ and $\boldsymbol{\mathcal{M}}_2$, for example originating from two models or originating from a model and from amplitude extraction based on real data, one can define

$$\mathcal{D}\left[\boldsymbol{\mathcal{M}}_1, \boldsymbol{\mathcal{M}}_2\right] = \arccos \Re \left(\boldsymbol{\mathcal{M}}_1^\dagger \boldsymbol{\mathcal{M}}_2 \right) . \tag{12}$$

Equation (12) depends on the choice for the reference phase. To get rid of this dependence, we minimize the distance between the amplitudes sets by varying the reference phase of one of the amplitude vectors, while keeping the other one fixed

$$\alpha_4 = \underset{\alpha_4}{\operatorname{argmin}} \left(\mathcal{D}\left[\boldsymbol{\mathcal{M}}_1(\alpha_4), \boldsymbol{\mathcal{M}}_2(\alpha_4' = 0) \right] \right) . \tag{13}$$

Inserting this result into Eq. (12) yields a reference-phase independent distance measure in amplitude space.

At any given $(W, \cos\theta_{\text{c.m.}})$, one can calculate the distance between two models in amplitude space. In order to be able to tell two models apart with a given a set of polarization measurements, we require that the standard deviation on the extracted reaction amplitudes is smaller than the typical distance between the two models. A bootstrap algorithm is a reliable and robust way to determine realistic error bars on the extracted amplitudes. The algorithm, for example, takes into account the non-linear relations between the amplitudes and the observables. The combination of observables at a particular $(W, \cos\theta_{\text{c.m.}})$, yields a set of asymmetries $\{A_i \pm \Delta A_i \,|\, A_i \in \text{set of observables}\}$, resulting in the amplitudes $\boldsymbol{\mathcal{M}}$. One generates an ensemble of N observable sets by sampling a normal distribution with $(\mu = A_i, \sigma = \Delta A_i)$ for each observable in the set. This results in an ensemble of amplitudes $\{\boldsymbol{\mathcal{M}}_{i=1,\ldots,N}\}$. The sample's standard deviation

$$\Delta\boldsymbol{\mathcal{M}} = \sqrt{\sum_{i=1}^{N} \frac{\mathcal{D}\left[\boldsymbol{\mathcal{M}}_i, \boldsymbol{\mathcal{M}}\right]^2}{N}} , \tag{14}$$

of the latter is an estimator for the error bars on $\boldsymbol{\mathcal{M}}$.

The left panel of Fig. 2 shows the $(W, \cos\theta_{\text{c.m.}})$ dependence of the distance between the RPR-2011 model and its Regge background. The largest difference between these two model variants is observed at backward $\theta_{\text{c.m.}}$, where the resonance content dominates. At forward $\theta_{\text{c.m.}}$ the reaction is predicted to be dominated by the background. The right panel shows the angular dependence of $\Delta\boldsymbol{\mathcal{M}}$, at fixed W. The result indicated as PD is obtained from pseudo data with error bars which are in line with those from published CLAS and GRAAL data. The figure also shows (lines indicated as UD) the anticipated resolution after adding pseudo data for the double-polarization observables that are currently under analysis. Instead of directly comparing data with models, we quantify the resolution of the data as a function of the kinematics and use two models to discover the kinematical regions of particularly large or small amplitude distances. Figure 2 shows that overcomplete sets will provide a stringent test for models. This is due to the fact that the double-polarization data provide a large amount of information on the δ_i and that the predictions of those vary enormously from one model to another.

4. Conclusions

We have shown that in the $p(\gamma, K^+)\Lambda$ reaction, the multipoles receive large contributions from the background diagrams. Hence, the extracted resonance content depends heavily on the theoretical framework applied to model the background. We propose amplitude extraction as a method for constraining model assumptions with regard to the description of the background. More in particular, amplitude extraction has a great potential to discriminate between different model hypotheses. To this end, we quantified the resolution of published and upcoming data in amplitude space. Simulations of amplitude extractions with realistic pseudo data illustrate how the novel polarization data will enormously affect this resolution and enable one to further constrain the underlying dominant reaction mechanisms.

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