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Review

The generalized parton distribution of the pion in a relativistic Bethe–Salpeter quark model

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Abstract

We present a Poincaré covariant calculation of the generalized parton distribution of the pion. Results for different values of the kinematical parameters are shown and discussed.

Keywords: Hadron physics; Bethe–Salpeter models; Generalized parton distributions

In the last decade, generalized parton distributions (GPDs) have played a key role in the exploration of hadron structure. We will focus on the vector GPD of the pion. For spin $1/2$ partons, only one such GPD exists, in contrast with the four GPDs of the nucleon. Therefore, the pion is a good choice for an exploratory model calculation.

The pion's quark GPD is defined as follows [1]:

$$H_\pi^q(x,\xi,t) = \frac{1}{2}\int \frac{\mathrm{d}z^-}{2\pi} \mathrm{e}^{\mathrm{i}x\bar{P}^+z^-} \left.\langle \bar{P}'|\bar{\psi}^q\left(-\frac{z}{2}\right)\gamma^+\psi^q\left(\frac{z}{2}\right)|\bar{P}\rangle\right|_{z^+=z_\perp=0}. \tag{1}$$

It can be shown that the skewedness ξ is constrained to the interval $[0,\xi_{\max}]$ with $\xi_{\max} = \sqrt{\frac{-t}{4M_\pi^2-t}}$, while x is constrained to the interval $[-\xi,1]$ for quark GPDs and $[-1,\xi]$ for antiquark GPDs. Outside these x intervals, the GPD should vanish. A model which resolves these intervals and yields a zero GPD otherwise, is said to have the correct support.

We adopt the Poincaré covariant constituent quark model which was developed at the University of Bonn [2]. The starting point of this model is the Bethe–Salpeter equation for a quark–antiquark system, bound by a confinement interaction and the 't Hooft instanton induced interaction. The latter works as a residual interaction, and is responsible for the strong binding of the pion. The propagators of the constituent quark and antiquark are approximated by Feynman propagators for spin $1/2$ particles with a constituent mass. Moreover, the interactions do not depend on

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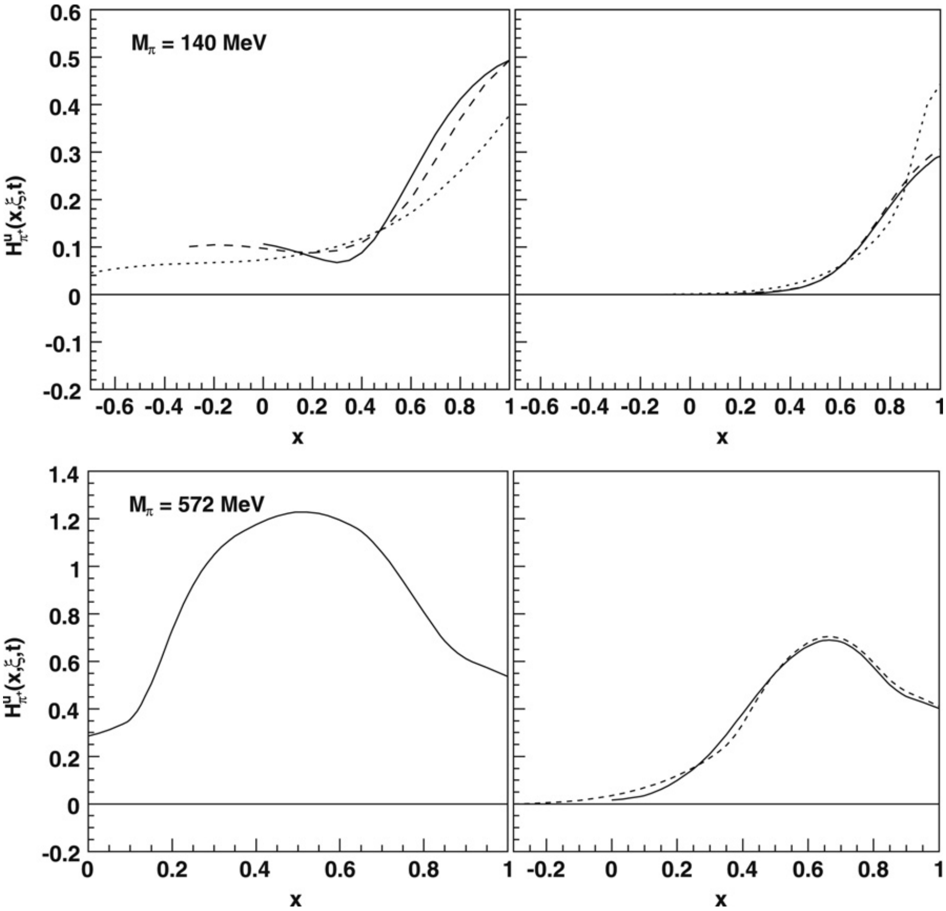

Fig. 1. The generalized parton distribution for $M_\pi = 140$ MeV (top) and for $M_\pi = 572$ MeV (bottom). Values for t are $t = -0.1$ GeV^2 (left) and $t = -1.0$ GeV^2 (right). The full line corresponds to $\xi = 0$, the dashed line to $\xi = 0.3$ and the dotted line to $\xi = 0.7$.

the relative energy variables in the hadron rest frame (instantaneous approximation). Within this model, one is able to calculate static as well as dynamic quantities for the full $q\bar{q}$ mesonic spectrum.

Any model for the GPD of the pion will have to fulfill the sum rule for the electromagnetic form factor, $\int \mathrm{d}x\, H_\pi^q(x,\xi,t) = F_\pi^q(t)$, and the isospin symmetry relation $H_\pi^q(x,\xi,t) = -H_\pi^{\bar{q}}(-x,\xi,t)$. As the quark model under study may suffer from a support violation [3], the following support parameter is defined for the quark GPD as a measure for the relative violation:

$$\phi = \frac{\int_{-\xi}^{1} |H_\pi^q(x,\xi,t)|\mathrm{d}x}{\int_{-\infty}^{\infty} |H_\pi^q(x,\xi,t)|\mathrm{d}x}. \tag{2}$$

Results for H_π^q in our model are shown in Fig. 1. We adopt the full model and the model without the residual 't Hooft interaction. The latter gives a pion mass of $M_\pi = 572$ MeV. Values for t are $t = -0.1$ GeV^2 (left pictures) and $t = -1.0$ GeV^2 (right pictures).

Our numerical results point out that the isospin symmetry relation $H_\pi^q(x,\xi,t) = -H_\pi^{\bar{q}}(-x,\xi,t)$ is exactly fulfilled, hence Fig. 1 only shows the quark GPDs. Furthermore, the sum rule $\int_{-\infty}^{\infty} \mathrm{d}x\, H_\pi^q(x,\xi,t) = F_\pi^q(t)$ for the GPD indeed yields the electromagnetic form factor for all ξ. However, the integration domain already indicates that the support regions in x are not resolved. A closer look at the values of the support parameter of Eq. (2) for the curves of Fig. 1 shows that increasing $|t|$ results in a better support (for $\xi = 0$: $\phi = 0.12$ at $t = -0.1$ GeV^2 versus $\phi = 0.19$ at $t = -1$ GeV^2). The largest effect is caused by the binding energy: the smaller the binding energy, the better the support ($\phi = 0.64$ at $t = -0.1$ GeV^2). Bearing in mind that the constituents have a mass of 380 MeV, it is seen that the binding energy for the pion with a mass of $M_\pi = 572$ MeV approaches the binding energy in the nucleon. As a consequence, we expect support to be much better for the nucleon than for the (physical) pion.

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