

**Inclusive and pion production neutrino-nucleus cross sections**

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We analyze the experimental data on the inclusive double-differential cross section by neutrinos charged current, measured by T2K, with the same model which was successful for the MiniBooNE quasielastic cross sections. As in our previous analysis the multinucleon component is needed in order to reproduce the data. For the total cross section, our evaluation is smaller than the SciBooNE data above 1 GeV. This indicates the opening of a new channel not included in our evaluation, presumably the two-pion-emission channel. We also check that our description holds for the exclusive single-pion-production channel by confronting our evaluation with the MiniBooNE double-differential cross section for a single charged pion and the  $Q^2$  distribution. Both are compatible with the data.

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**I. INTRODUCTION**

Many data on the cross sections of neutrinos or antineutrinos with light nuclei, namely  $^{12}\text{C}$ , are now available [1–8]. In addition to extensive MiniBooNE results [1–3,5,6,8], the T2K collaboration has issued data on the inclusive double-differential cross section [7]. The investigation of this quantity is interesting because it brings another test for the validity of a theoretical description. Indeed, the T2K neutrino beam [9] is different from the MiniBooNE one [10]: it peaks at similar energies,  $E_\nu \simeq 600$  MeV, as the MiniBooNE one but it is definitely narrower and closer to a monochromatic beam. Thus the analysis of the T2K data offers another test for our description. The differential cross section measured by T2K incorporates pion production. In our work of Ref. [11], we calculated the quasielastic channel, the multinucleon channel, and the single-pion (coherent and incoherent) emission. Assuming that these are the only channels involved in the experiment, the T2K data are linked to our total response. We also discuss in this work the MiniBooNE data on the double-differential partial cross section for single-pion production [5].

One interesting aspect of these data is the fact that in the angular distributions one bin corresponds to small angles for muon emission,  $0.95 < \cos\theta < 1$  for MiniBooNE and  $0.94 < \cos\theta < 1$  for T2K. The measurement of the forward cross section offers a chance of access to the elusive isospin spin-longitudinal response which is of a particular interest due to its collective aspects with the presence in particular of the coherent-pion-production channel. The isospin spin-transverse response, where coherent-pion production is essentially absent, quickly dominates when one departs from the forward direction.

In this work we use the same model which has been successful for the MiniBooNE data on the neutrino and antineutrino quasielastic-like cross sections, the total cross sections, or the double-differential cross sections, as shown in Refs. [11–14]. We summarize here the basic ingredients of

this model which is based on the nuclear response functions. In our description, the quasielastic response is treated in the random phase approximation (RPA), as discussed by Alberico *et al.* in Ref. [15]. For the isospin spin-transverse response the particle-hole force is repulsive and its main effect is a hardening effect and a quenching effect due to the mixing of nucleon-hole states with  $\Delta$ -hole states: the Ericson-Ericson-Lorentz-Lorenz effect [16]. The multinucleon contribution is evaluated as in our previous articles [11–14]. It is deduced from the microscopic calculation of Alberico *et al.* [17] on the role of two-particle-two-hole (2p-2h) contribution in the inclusive  $(e, e')$  transverse response. This calculation includes the correlation term, the two-body exchange terms, in particular the one associated with  $\Delta$  excitation, and the interference between these quantities. As for the single-pion production, we assume, as previously done [11], that it arises exclusively from the pionic decay of the  $\Delta$  excitation. In the nucleus the  $\Delta$  width is reduced by medium effects such as the nonpionic  $\Delta$  decay which leads to 2p-2h or 3p-3h excitations; they have been introduced and discussed by Oset and Salcedo in Ref. [18]. We use their parametrization for the in-medium  $\Delta$  width. The nonpionic decay of the  $\Delta$  in the medium contributes to our  $n$ -particle- $n$ -hole ( $np$ - $nh$ ) cross section.

**II. INCLUSIVE CROSS SECTION**

We first discuss the T2K results [7]. The T2K cross section for charged currents (CCs) does not isolate specific channels but sums over all accessible final states. For the comparison with these data we assume that the only channels opened are the quasielastic, the multinucleon emission and the single-pion production. Notice that here, in contradistinction with the pion-emission case, the final-state interaction of the emitted pion which depopulates the pion channel populates multinucleon states and therefore does not reduce the inclusive cross section. Figure 1 displays our prediction for the double-differential cross section as function of the emitted muon

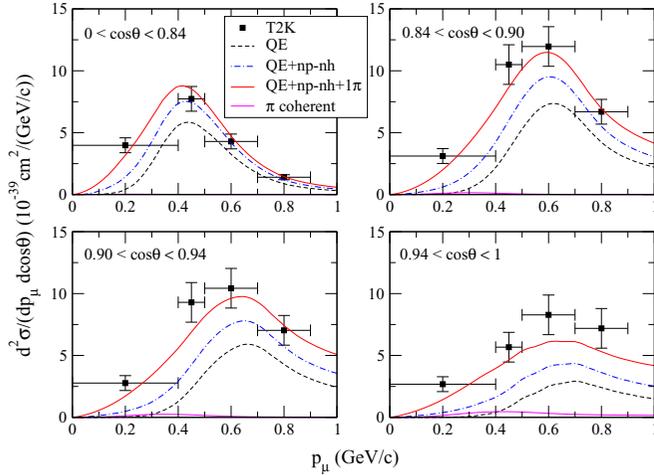


FIG. 1. (Color online) T2K flux-averaged inclusive CC double-differential cross section on carbon per nucleon as a function of muon momentum. The different contributions to this inclusive cross section obtained in our model are shown. The experimental T2K points are taken from Ref. [7].

momentum for the various angular bins. The experimental points are the T2K measured ones. We show separately the different components of the theoretical cross section: first the genuine quasielastic channel, second the total quasielastic-like one including the multinucleon component, and the total one including the single-pion-production cross section. The separate contributions are given in order to allow future comparisons with analysis of the quasielastic channel in T2K which are in progress [19]. The coherent-pion-production component is also shown in Fig. 1 but in this inclusive cross section its contribution is too small to be singled out. Our evaluation is compatible with the data. As in our previous analysis of the MiniBooNE quasielastic-like cross sections the multinucleon component is needed in order to reproduce the experimental results.

For the smallest-angle bin some underevaluation in the theory seems to show up. In this respect we can make the following comment: The forward direction, which corresponds to  $q \simeq \omega$ , is special in one important aspect: the spin transverse and the charge (isovector) contributions are kinematically suppressed and only the spin longitudinal one survives [20]. For small or moderate  $q$  values, this last response includes two separated regions of response, one at relatively large energy transfers,  $\omega > m_\pi$ , and one at low energy with the quasielastic component. In addition in nuclei the  $np$ - $nh$  response fills all the  $(\omega, q)$  plane. The large-energy part contributes to pion emission, coherent or not, and to multinucleon emission. They are included in our predictions. The contribution of the low-energy part, in the quasielastic region, should in principle be important. However, in the evaluation of the spin longitudinal contribution there appears a factor  $[\omega - Q^2/(2M)]$ , which vanishes identically for the quasielastic kinematics [11,21]. Strictly speaking, this cancellation is true for a nucleon initially at rest, but in practice it remains true also in the Fermi gas. Indeed, our numerical evaluation of the spin longitudinal quasielastic contribution in neutrino or

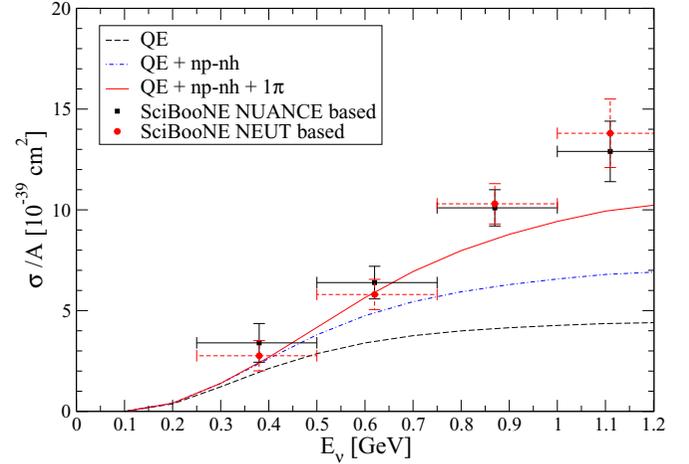


FIG. 2. (Color online) Inclusive CC cross section on carbon per nucleon as a function of neutrino energy. The experimental SciBooNE points are taken from Ref. [4].

antineutrino interactions (which is the same in both cases) shows its smallness for all neutrino energies, as illustrated in Fig. 3 of our previous work [12]. One can also observe in Fig. 1 that the quasielastic contribution is smaller for the smallest-angle bin. In view of these cancellations, one is led to consider other contributions beyond the quasielastic kinematics in order to avoid the canceling effect. This is, for instance, the case for the excitation of collective giant resonances. Their energy is low,  $\sim 10$  to  $30$  MeV, which is small compared to the neutrino energy, some hundreds of MeV. The small energy transfer in their excitation implies that the muon energy is nearly the same as the neutrino energy, a few hundreds of MeV, the region where the excess of the experimental cross section seems to occur. At large angles the contribution of the collective states is suppressed by form-factor effects [22]. Several studies have been made on the excitation of low-energy collective states in neutrino interactions [22–28] but specific work is needed to assess their role in the present type of data where forward bins offer favorable conditions to display their contribution.

For the inclusive cross section as a function of the neutrino energy, experimental results have been previously published by the SciBooNE collaboration [4]. We report them in Fig. 2 together with our theoretical prediction which gives a good fit of the data up to  $E_\nu \simeq 1$  GeV but underestimates the cross section above this value, as also reported by Nieves *et al.* [30]. The natural interpretation is the existence of other channels which open up at high energies and which have not been included in our analysis. A likely candidate for the missing channel is the multi-pion production, in particular the two-pion production channel, as also suggested in Refs. [30,31]. As an illustration of the likely importance of this channel and although it has no direct connection to the neutrino cross section, we report in Fig. 3 the total photoabsorption cross section by a proton as a function of photon energy, as well as the cross sections for the exclusive channels: one-pion-production and two-pion-production channels taken from the

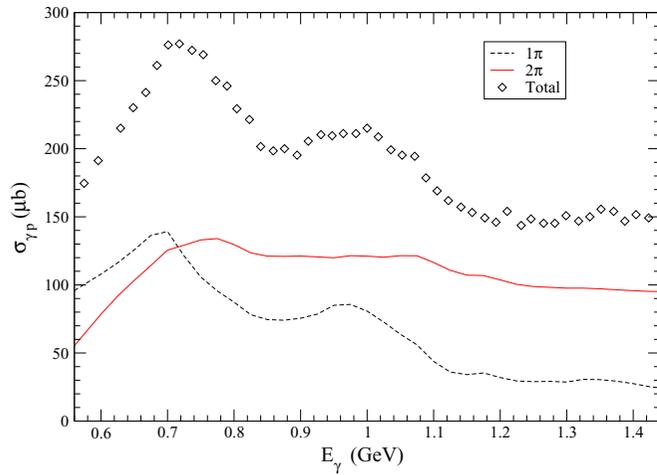


FIG. 3. (Color online) Total photoabsorption cross section for free proton as a function of photon energy measured at GRAAL 2008. The points are taken from Ref. [29]. The one-pion production and two-pion production contributions are separately plotted.

GRAAL experimental results [29]. Beyond a photon energy of about  $E_\gamma \simeq 0.7$  GeV, the two-pion-production cross section dominates over the single-pion cross section and it represents an important part of the total cross section. For incident pions as well, the two-pion-production cross section which has been studied by Oset and Vicente Vacas [32] becomes important for energies above the  $\Delta$  resonance and it increases with energy.

For neutrinos as well, one can expect a similar behavior with a dominance of the two-pion emission as compared to that of a single pion. The evaluation of the two-pion-production process by neutrinos has been studied by Hernandez *et al.* [33] but only close to the two-pion threshold. It should be extended at larger energies. A sizable two-pion component directly affects the inclusive cross section which sums over final states, and its omission is a likely candidate for the underevaluation of the total cross section by our theory at large neutrino energies. We also remind the reader of the possible contribution of deep inelastic scattering, recently examined in connection with the T2K results in Ref. [34].

### III. ONE-PION-PRODUCTION CROSS SECTION

In previous works [11–14] we investigated the quasielastic channel measured by MiniBooNE [1,8]. Here, we want to test our model, as described in Ref. [11], on the pion-production channel. In this section we compare the MiniBooNE data [5] on single-charged-pion production by neutrino reactions on mineral oil,  $\text{CH}_2$ , with our predictions based on our model of Ref. [11]. The results of the double-differential cross section as a function of the muon variables, emission angle, and energy (hence not affected by the neutrino energy reconstruction problem) are shown in Fig. 4. Our theoretical cross section for the molecule  $\text{CH}_2$  incorporates the two-hydrogen contributions which are free of nuclear effects. The general agreement between our evaluation and the data is good. The single-differential cross section as a function of the muon kinetic

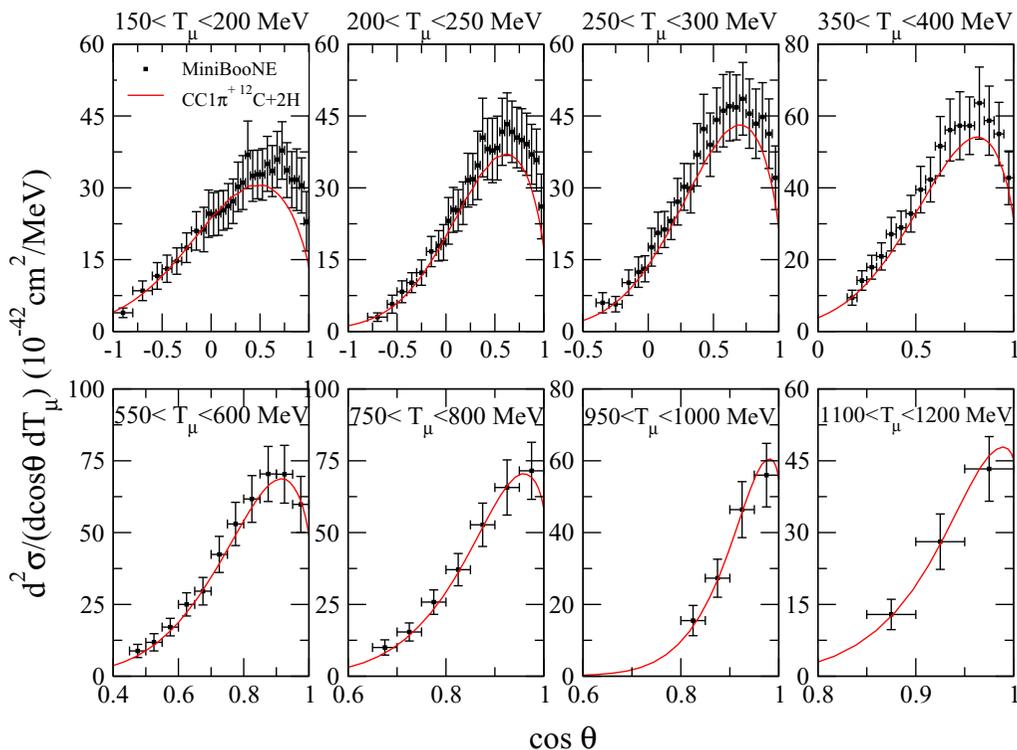


FIG. 4. (Color online) MiniBooNE flux-averaged CC  $1 \pi^+ \nu_\mu$ - $\text{CH}_2$  double-differential cross section for several values of muon kinetic energy as a function of the muon scattering angle. The experimental MiniBooNE points with the shape uncertainty are taken from Ref. [5].

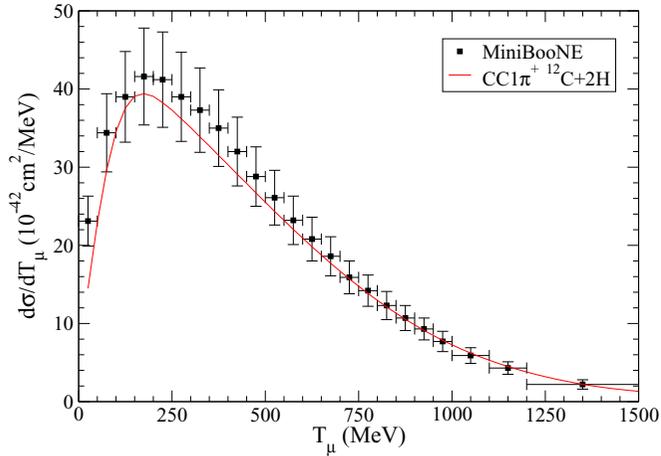


FIG. 5. (Color online) MiniBooNE flux-averaged CC  $1 \pi^+ \nu_\mu$ - $\text{CH}_2$  differential cross section as a function of muon kinetic energy. The experimental MiniBooNE points are taken from Ref. [5].

energy is shown in Fig. 5 and the  $Q^2$  distribution is shown in Fig. 6, in which the coherent contribution is also singled out. These quantities are also rather-well reproduced. As was stressed in Ref. [11], our model does not incorporate the final-state interaction (FSI) for the emitted pion on its way out of the nucleus which reduces the pionic cross section and which should lead to an overestimation of our theory as compared with the data. However, this difference does not show up in this comparison with data. The role of the final-state interaction has been discussed by several authors [35,36]. In their works [35,36], the MiniBooNE differential-cross-section function of the final pion momentum (a quantity that our approach does not calculate) is evaluated. Both works display a reshaping of this differential cross section due to the inclusion of the pion FSI. The inclusion of this distortion suppresses the agreement with the MiniBooNE data. Instead, for the processes of pion photoproduction and pion absorption, the inclusion

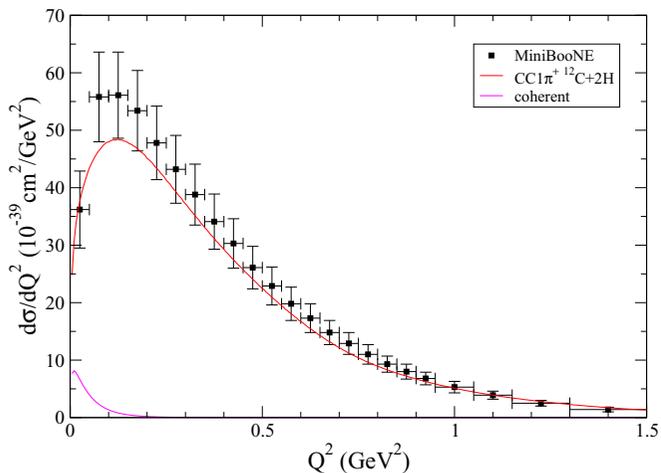


FIG. 6. (Color online) MiniBooNE flux-averaged CC  $1 \pi^+ \nu_\mu$ - $\text{CH}_2$   $Q^2$  distribution. The coherent channel is separately shown. The experimental MiniBooNE points are taken from Ref. [5].

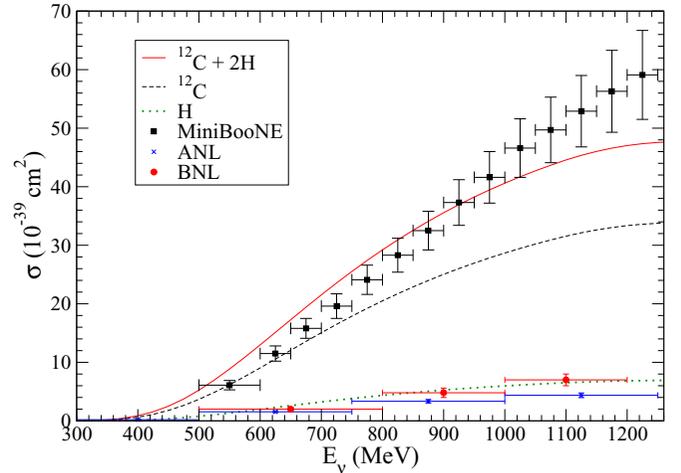


FIG. 7. (Color online) CC  $1 \pi^+ \nu_\mu$ - $\text{CH}_2$  total cross section as a function of the neutrino energy compared to the experimental MiniBooNE results [5]. Single-pion production on proton as well as ANL [39] and BNL [40] results are also given.

of pion FSI in the theoretical calculations is crucial in order to reproduce the data. The puzzle of the absence of a clear experimental manifestation of the influence of pion FSI in the MiniBooNE data was recently reviewed in Refs. [37,38].

The comparison of our total cross section with data for single-charged-pion production as a function of neutrino energy is displayed in Fig. 7. In this figure our hydrogen contribution, which agrees with the BNL data [40], is shown. The overall agreement is moderate, with an overevaluation of the theory at small energies and an underevaluation at large energies. No correction has been applied in Fig. 7 for the reconstruction of the neutrino energy, which may increase further the deviation of the theory from the data [41–45]. A similar question between the fits of the double-differential cross-section function of the muon variables and that of the integrated cross section is present in the description of Ivanov *et al.* [46]. At low neutrino energy, the fact that the data are below the predictions could be an effect of the pion final-state interaction which is not incorporated in our theoretical description. At larger neutrino energies the underevaluation by the theory could result from the two-pion production. This last process does not enter directly into the single-pion-production cross section but it could influence it through a misidentification phenomenon, if one of the two pions is absorbed in the nucleus on its way out. The observed deviation of the theory, which goes from overevaluation to underevaluation, would result from the two phenomena. The double-differential cross section instead is a flux-integrated quantity. It could be less sensitive if these two opposite effects partly canceled each other.

It is interesting to display the results for the most-forward bin for the muon angle  $0.95 < \cos \theta < 1$ . The double-differential cross section for this bin is shown as a function of muon kinetic energy in Fig. 8. Here, the coherent contribution is significant although not dominant. This contribution is interesting due to its relation to a high-energy collective state

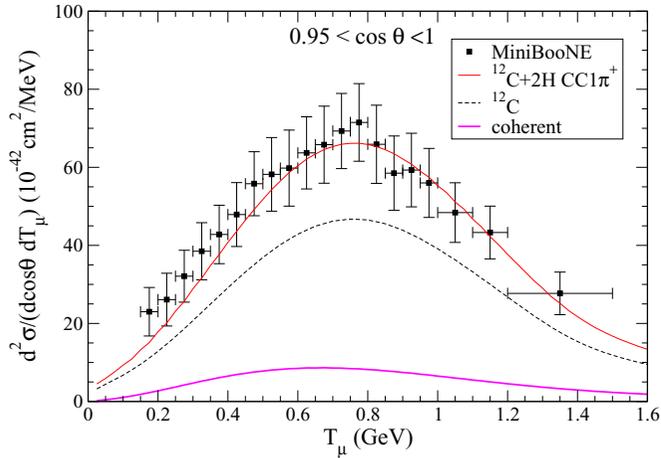


FIG. 8. (Color online) MiniBooNE flux-averaged CC  $1 \pi^+ \nu_\mu$ - $\text{CH}_2$  double-differential cross section for  $0.95 < \cos \theta < 1$  as a function of the muon kinetic energy. The coherent channel is separately shown. The experimental MiniBooNE points are taken from Ref. [5].

of the nucleus, denoted as the “pion branch” [47,48]. It is a coherent mixture of  $\Delta$ -hole states and pions, as was nicely illustrated in the two-level model of Delorme and Guichon [48]. In the  $(\omega, q)$  plane the pion branch, which embodies the modification of the dispersion relation for pion propagation in the nuclear medium by the polarization of the medium, i.e., by the virtual excitation of  $\Delta$ -hole states, sits at lower energies than the free-pion line. It can only show up for probes which have, as the pion, a spin longitudinal coupling,  $\vec{S} \cdot \hat{q}$ , i.e., along the momentum  $\vec{q}$ . In the interaction of physical pions with nuclei it shows up only indirectly since the energy-momentum relation, restricted to that of a physical pion,  $\omega^2 - \vec{q}^2 = m_\pi^2$ , is not that of the collective state. In this case one observes only the depletion due to the undetected collective state. The condition for its display in neutrino interactions was discussed by Delorme and Ericson [20]. They pointed out that, for neutrinos, it is only in the forward direction that the spin longitudinal response, which is sensitive to the pion branch, can dominate the cross section. This response contains the coherent-pion production which represents the emission of a physical pion by a  $\Delta$ -hole bubble, with the nucleus remaining in its ground state. Pion emission can also occur via a series of  $\Delta$ -hole bubbles, i.e., via the pionic decay of the pion branch in which the collective state transforms into a pion, as illustrated in Fig. 9. The signature for the collective pion branch is a

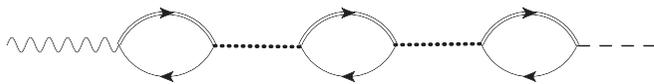


FIG. 9. Diagrammatic representation of the coherent-pion-production process. The wiggly line represents the external probe with the spin-longitudinal coupling. Double lines correspond to the propagation of a  $\Delta$ , solid lines correspond to the propagation of a nucleon hole, dotted lines correspond to the  $\Delta$ -hole interaction, and the dashed line represents the pion.

shift towards smaller energy transfer of the coherent-pion emission cross section with respect to the first-order term with only one  $\Delta$ -hole bubble. In the MiniBooNE experiment, the contribution of the coherent term is not sufficient to perform a quantitative study but this possibility can be envisaged for the future. Notice that, in analogy with the photoproduction of neutral and charged pions leading to discrete nuclear states [49,50], also for the neutrino interactions a contribution from low-energy excitations of  $^{12}\text{C}$  or  $^{12}\text{B}$  in the case of charged currents together with one-pion emission (a “quasicoherent” pion emission) can also be expected in the forward bins. It is not included in our description.

#### IV. CONCLUSION

In summary, we tested our model of the neutrino nucleus interaction on the T2K inclusive data and on the MiniBooNE single-pion-production cross section. For the double-differential cross sections which are free from neutrino-energy reconstruction problems, the agreement is generally satisfactory. The comparison with the T2K inclusive results represents the first successful test of the necessity of the multinucleon emission channel in an experiment with another neutrino flux with respect to the one of MiniBooNE. Even with the inclusion of the  $np$ - $nh$  excitations, some underevaluation by the theory of the T2K data seems to show up in the forward direction. It could be due to some contributions not included in our description, such as excitations of low-lying giant resonances. In the single-pion-production MiniBooNE data, the lowest-angle bin is sensitive to the coherent-pion-production cross section. Presently, the importance of this contribution is not sufficient to perform a detailed study of this interesting channel but, in the future, it could become accessible with some improvements in the angular resolution so as to be more concentrated on the forward direction. For the integrated cross sections, the underevaluation of our theory with respect to the data above an energy  $E_\nu \simeq 1$  GeV is presumably due to the two-pion-production process which influences the inclusive cross section directly, but also the single-pion exclusive cross section through a misidentification process if one of the two pions is absorbed. The theoretical description should be improved in this direction with the inclusion of the two-pion channel.

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