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Mass dependence of short-range correlations in nuclei and the EMC effect

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Abstract.

An approximate method to quantify the mass dependence of the number of two-nucleon (2N) short-range correlations (SRC) in nuclei is suggested. The proposed method relies on the concept of the "local nuclear character" of the SRC. We quantify the SRC and its mass dependence by computing the number of independent-particle model (IPM) nucleon pairs in a zero relative orbital momentum state. We find that the relative probability per nucleon for 2N SRC follows a power law as a function of the mass number A. The predictions are connected to measurements which provide access to the mass dependence of SRC. First, the ratio of the inclusive inelastic electron scattering cross sections of nuclei to ^{2}H at large values of the Bjorken variable. Second, the EMC effect, for which we find a linear relationship between its magnitude and the predicted number of SRC-prone pairs.

1 Introduction

In a mean-field model fluctuations are completely ignored. The SRC induce spatio-temporal fluctuations from the mean-field predictions. Realistic nuclear wave functions reflect the coexistence of single-nucleon (mean-field) structures and cluster structures. The clusters account for beyond mean-field behavior. As the nucleon-nucleon interaction is short ranged, the clusters attributed to SRC are predominantly of the two-nucleon (2N) type. Given an arbitrary nucleus $A(N, Z)$ we address the issue of quantifying the number of SRC-prone pairs. Our suggested method, albeit approximate, is robust, model independent, and is applicable to any nucleus from He to Pb. Our goal is to come with a systematic insight into the mass and isospin dependence of the SRC without combining results from various types of calculations.

2 Quantifying nuclear correlations

2.1 Mean-field approximation and beyond

A time-honored method to account for the effect of correlations in classical and quantum systems is the introduction of correlation functions. Realistic nuclear wave functions $\mid \Psi_A\rangle$ can be computed after applying a many-body correlation operator to a Slater determinant $\mid \Psi_A^{MF}\rangle$

$$\mid \Psi_A\rangle = \frac{1}{\sqrt{\langle \Psi_A^{MF} \mid \widehat{\mathcal{G}}^\dagger \widehat{\mathcal{G}} \mid \Psi_A^{MF} \rangle}} \widehat{\mathcal{G}} \mid \Psi_A^{MF} \rangle \; . \tag{1}$$

The nuclear correlation operator $\widehat{\mathcal{G}}$ has a complicated spin, spin-orbit and isospin dependence but is dominated by the central, tensor and spin correlations [1]

$$\begin{aligned} \widehat{\mathcal{G}} &\approx \widehat{\mathcal{S}}\Big[\prod_{i<j=1}^{A}\Big(1 - g_c(r_{ij}) + f_{t\tau}(r_{ij}) S_{ij} \vec{\tau}_i \cdot \vec{\tau}_j \\ &+ f_{s\tau}(r_{ij}) \vec{\sigma}_i \cdot \vec{\sigma}_j \;\; \vec{\tau}_i \cdot \vec{\tau}_j \Big)\Big] \; , \end{aligned} \tag{2}$$

where g_c, $f_{t\tau}$, $f_{s\tau}$ are the central, tensor, and spin-isospin correlation function, $\widehat{\mathcal{S}}$ the symmetrization operator, S_{ij} the tensor operator, and $\vec{r}_{ij} = \frac{\vec{r}_i - \vec{r}_j}{\sqrt{2}}$. The operator S_{ij} admixes relative two-nucleon states of different orbital angular momentum, is operative on triplet spin states only, and conserves the total angular momentum of the pair.

The effect of the correlation functions on the momentum distributions can be roughly estimated from their squared Fourier transforms. The relative momentum ($\vec{k}_{12} = \frac{\vec{k}_1 - \vec{k}_2}{\sqrt{2}}$) dependence of squared Fourier transform of the tensor correlation $\mid f_{t\tau}(k_{12}) \mid^2$ is very similar to the squared D-wave component of the deuteron wave function $\mid \Psi_D(k_{12}) \mid^2$ [2]. The effect of the tensor correlation function is largest for moderate relative momenta ($100 \lesssim k_{12} \lesssim 500$ MeV). For very large k_{12}, the g_c is the dominant contribution. Whereas a large model dependence for the g_c is observed, the $f_{t\tau}$ seems to be much better constrained.

After introducing the wave functions of Eq. (1), the one-body momentum distributions can be written as

$$P_1\left(\vec{k}\right) = P_1^{(0)}\left(\vec{k}\right) + P_1^{(1)}\left(\vec{k}\right) \; . \tag{3}$$

The $P_1^{(0)}$ is the mean-field part and is fully determined by the Slater determinant $\mid \Psi_A^{MF} \rangle$. The SRC generate a fat momentum tail to the $P_1\left(\vec{k}\right)$. The high momentum tails to $n_1^{(1)}(k) = \int \mathrm{d}\Omega_k P_1^{(1)}(\vec{k})$ have a very similar momentum dependence for all nuclei, including the deuteron, which alludes to an universal character of SRC [3]. It has been theoretically predicted [4–6] and experimentally confirmed in semi-exclusive $A(e, e'p)$ measurements [7] that the major fraction of the high-momentum tail to $n_1^{(1)}(k)$ strength is contained in very specific parts of the single-nucleon removal energy-momentum phase space, namely those where the ejected nucleon is part of a pair with high relative and small c.m. momentum, the so-called ridge in the spectral function [5].

2.2 Quantifying two-nucleon correlations

Theoretical $^{16}\mathrm{O}(e, e'pn)$ calculations [5,8,9] have predicted that the tensor parts of the SRC are responsible for the fact that the correlated $(e, e'pn)$ strength is typically a factor of 10 bigger than the correlated $(e, e'pp)$ strength. Calculations indicated that the tensor correlations are strongest for pn pairs with "deuteron-like" $|l_{12} = 0, S = 1\rangle$ relative states [8,9]. Recently, this dominance of the pn correlations over pp and nn ones has been experimentally confirmed [10,11].

Accordingly, a reasonable estimate of the amount of correlated nucleon pairs in $A(N, Z)$ is provided by the number of pairs in a $l_{12} = 0$ state. In order to determine that number for a given set of single-particle states, one needs a coordinate transformation from $(\vec{r}_1, \vec{r}_2)$ to $\left(\vec{r}_{12} = \frac{\vec{r}_1 - \vec{r}_2}{\sqrt{2}}, \vec{R}_{12} = \frac{\vec{r}_1 + \vec{r}_2}{\sqrt{2}}\right)$. For a harmonic oscillator (HO) Hamiltonian this transformation can be done with the aid of Moshinsky brackets [12]. After introducing the spin and isospin degrees-of-freedom, in a HO basis a normalized and antisymmetrized (na) two-nucleon state reads ($\alpha_i \equiv (n_i l_i j_i t_i)$)

$$
\begin{aligned}
&|\alpha_1\alpha_2; JM\rangle_{na} = \frac{(1-\mathcal{P}_{12})}{\sqrt{2\,(1+\delta_{\alpha_1\alpha_2})}}\,|\alpha_1\,(\vec{r}_1)\,\alpha_2\,(\vec{r}_2)\,; JM\rangle \\
&= \sum_X \langle n_{12} l_{12} N_{12} \Lambda_{12} L M_L, S M_S T M_T \mid \alpha_1\alpha_2; J_M J\rangle \\
&\times \left|\left[n_{12} l_{12}\,(\vec{r}_{12}), N_{12}\Lambda_{12}\left(\vec{R}_{12}\right)\right] L M_L, S M_S, T M_T\right\rangle \ ,
\end{aligned}
\tag{4}
$$

where $\sum_X$ sums over the appropriate quantum numbers (n_{12}, l_{12}, N_{12}, Λ_{12}, L, M_L, S, M_S, T, M_T), $\langle \ldots \mid \ldots \rangle$ is the transformation bracket as defined in Ref. [2] and $\mathcal{P}_{12}$ the interchange operator for the spatial, spin, and isospin coordinate. Starting from the Eq. (4) one can compute in a HO single-particle basis how much a pair wave function with quantum numbers

$$
\left|\left[n_{12} l_{12}\,(\vec{r}_{12})\,, N_{12}\Lambda_{12}\left(\vec{R}_{12}\right)\right] L M_L, S M_S, T M_T\right\rangle \tag{5}
$$

contributes to the sum-rule

$$
\begin{aligned}
&\sum_{JM} \sum_{\alpha_1 \leq \alpha_F^{N_1}} \sum_{\alpha_2 \leq \alpha_F^{N_2}} {}_{na}\langle \alpha_1\alpha_2; JM\,|\alpha_1\alpha_2; JM\rangle_{na} \\
&= \begin{cases} \frac{N(N-1)}{2} & N_1 = N_2 = n \\ \frac{Z(Z-1)}{2} & N_1 = N_2 = p \\ NZ & N_1 \neq N_2 \end{cases} .
\end{aligned}
\tag{6}
$$

This can also be done for any other non-relativistic basis $|nljm\rangle$ of single-particle states in a two-step procedure. First, a 2N state can be expressed in

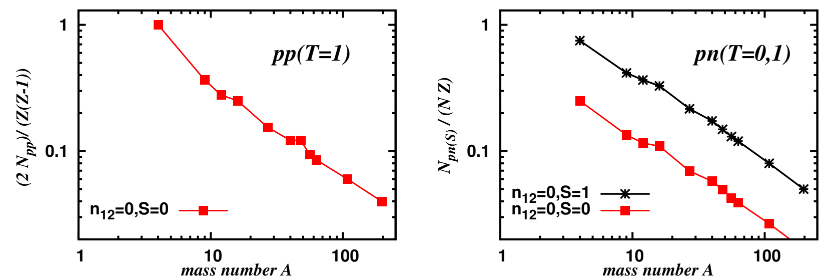

Figure 1. The computed values for $\frac{2}{Z(Z-1)}N_{pp}$, and $\frac{1}{(NZ)}N_{pn(S)}$ which represent the predicted fraction of the pairs which are prone to SRC. The results are obtained for HO single-particle wave functions with $\hbar\omega$ (MeV) $= 45A^{-\frac{1}{3}} - 25A^{-\frac{2}{3}}$ and for the target nuclei ^{4}He, ^{9}Be, ^{12}C, ^{16}O, ^{27}Al, ^{40}Ca, ^{48}Ca, ^{56}Fe, ^{63}Cu, ^{108}Ag, and ^{197}Au. The computed values for nn correlations are similar to the pp results and can and be found in Ref. [2].

a HO basis. Second, the Eq. (4) can be used to determine the weight of the pair wave functions of Eq. (5). The number of ($n_{12} = 0$, $l_{12} = 0$) pairs, which are stated to be a reasonable estimate for the number of the SRC-prone is given by the expression,

$$N_{pp}(A,Z) = \sum_{JM} \sum_{\alpha_1 \leq \alpha_F^p} \sum_{\alpha_2 \leq \alpha_F^p} {}_{na} \left\langle \alpha_1 \alpha_2; JM \right| \mathcal{P}_{\vec{r}_{12}}^{n_{12}=0 l_{12}=0} \left| \alpha_1 \alpha_2; JM \right\rangle_{na} \ , \tag{7}$$

where $\mathcal{P}_{\vec{r}_{12}}^{n_{12}=0 l_{12}=0}$ is a projection operator for 2N relative states with $n_{12} = 0$, $l_{12} = 0$. A similar expression to Eq. (7) holds for the nn pairs. For the pn pairs it is important to add the projection operator $\mathcal{P}_{\vec{\sigma}}^{S}$ to discriminate between the triplet and singlet spin states. In Fig. 1 we display some computed results for the N_{pp} and $N_{pn(S)}$ for 11 nuclei covering the full mass table. Naively one could expect that the number of correlated pn (pp) pairs in a nuclei scales like $NZ\left(\frac{Z(Z-1)}{2}\right) \sim A^2$. From Fig. 1 it is clear that the mass dependence of the number of SRC pairs approximately follows a power law. The pn ($n_{12} = 0$, $l_{12} = 0$, $S = 1$) SRC-prone pairs scale as $\sim A^{1.35 \pm 0.03}$.

The results for N_{nn} which are similar to N_{pp} can be found in Ref. [2]. In Ref. [2] a method to estimate the number of correlated $3N$ clusters is developed. We find that there is (as for 2N correlations) a power law relation between the mass A and the number of correlated ppn triples.

3 Results

3.1 Two-body correlations

Following the experimental observation [13–15] that the ratio of the inclusive electron scattering cross sections from a target nucleus A and from the deuteron

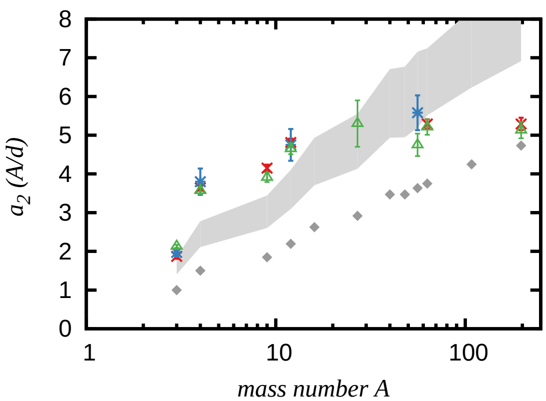

Figure 2. The computed values for the $a_2(A/D)$ for various nuclei. The data are from Refs. [14–16]. The shaded region is the prediction after correcting the computed values of $a_2(A/D)$ for the c.m. motion of the pair. The correction factor are determined by linear interpolation of the factors listed in Table 1 of Ref. [2]. The width of the shaded area is determined by the estimated errors of the c.m. correction factors.

D

$$\frac{\sigma^A\left(x_B,Q^2\right)}{\sigma^D\left(x_B,Q^2\right)}\ , \tag{8}$$

scales for $1.5 \lesssim x_B \lesssim 2$ and moderate Q^2, it has been suggested [13] to parameterize the ratio σ^A/σ^D in the following form

$$a_2\left(A/D\right) = \frac{2}{A}\frac{\sigma^A\left(x_B,Q^2\right)}{\sigma^D\left(x_B,Q^2\right)} \quad \left(1.5 \lesssim x_B \lesssim 2\right)\ . \tag{9}$$

In a simplified reaction-model picture, which ignores for example the effect of c.m. motion of pairs in finite nuclei, the quantity $\frac{A}{2}a_2\left(A/D\right)$ can be connected with the number of correlated pairs in the nucleus A [14]. Assuming that all pn pairs contribute one would expect that for the relative amount of correlated two-nucleon clusters $a_2\left(A/D\right) \sim A$. We suggest in Refs. [2, 17] that the correlated pn pairs contributing to the $a_2(A/D)$, are predominantly $(T=0, S=1)$ pairs and that $a_2(A/D)$ is proportional to the per nucleon probability for a pn SRC relative to the deuterium. Thereby, the per nucleon probability for a pn SRC relative to the deuterium can be defined as

$$\frac{2}{N+Z}\frac{N_{pn(S=1)}(A,Z)}{N_{pn(S=1)}(A=2,Z=1)} = \frac{2}{A}N_{pn(S=1)}(A,Z)\ . \tag{10}$$

Apart from corrections stemming from final-state interactions, a correction factor which accounts for the c.m. motion of the correlated pairs blurs the connection between the measured $a_2(A/D)$ coefficients and the number of correlated pairs. We have opted to correct the predicted a_2 coefficients and not the data for c.m. motion. The magnitude of the c.m. motion correction factor is subject of ongoing discussions [16] and is far from established. We stress that the c.m. correction factor cannot be computed in a model-independent fashion. To estimate the c.m. correction factor, we have simulated the number of events in the probed phase with and without accounting for pair c.m. motion. We simulate the interaction of a virtual photon with a nucleon pair inside a nucleus. We assume that the virtual photon reacts instantly with one of the nucleons inside the pair, i.e. the virtual photon is entirely absorbed by one of the paired nucleons. In Ref. [2] we stated a c.m. correction factor of 1.7 ± 0.3 which shows little mass dependence. For light nuclei our predictions corrected for c.m. motion of the pairs, underestimate the measured a_2. This may be attributed to the lack of long-range clustering effects in the adopted wave functions. Indeed, it was pointed out in Ref. [18] that the high-density cluster components in the wave functions are an important source of correlation effects beyond the mean-field approach. For heavy nuclei our predictions for the relative SRC probability per nucleon do not saturate as much as the data seem to indicate. We stress that final-state interactions (FSI) represent another source of corrections which may induce an additional A-dependent correction to the data. FSI of the outgoing nucleons with the residual spectator nucleons, could shift part of the signal's strength out (or, in) of the cuts applied to the experimental phase space and decrease (or increase) the measured cross section and the corresponding a_2 coefficient.

3.2 EMC effect

In 1983, the EMC collaboration discovered that the ratio $R(x_B)$ of the Deep Inelastic Scattering (DIS) cross section of leptons on a nucleus and the deuteron differs from one [19]. At medium Bjorken x_B-values, $0.3 \leq x_B \leq 0.7$, $R(x_B)$ drops from approximately one to values as low as 0.8. This effect is known as the EMC effect. This reduction of $R(x_B)$ is not easily explained and so far there still is no established explanation yet. More recently, a linear relation between the slope of the EMC effect $-\frac{\mathrm{d}R}{\mathrm{d}x_B}$ in the region $0.3 \leq x_B \leq 0.7$, and the SRC scaling factor $a_2(A/D)$, obtained from inclusive electron scattering, has been found [20]. Consequently, one may expect that $-\frac{\mathrm{d}R}{\mathrm{d}x_B}$ could be related to the number of SRC-prone pairs in the nucleus. When computing the $a_2(A/D)$ coefficients we included the SRC-prone $(S=1, T=0)$ pn pairs. This is justified by the dominance of the tensor correlation in the inclusive electron scattering data at moderate momentum transfers and high x_B. In the DIS experiments, which are performed at considerably higher Q^2, partons are the relevant degrees of freedom and one may argue that all correlated 2N pairs should be counted equally. Therefore when relating $-\frac{\mathrm{d}R}{\mathrm{d}x_B}$ to the number of correlated pairs, one should count all SRC-prone pairs including the $(S=0, T=1)$ pairs.

In Fig. 3 we display the magnitude of the EMC effect, quantified by means of $-\frac{dR}{dx_B}$ versus our predictions for the "per nucleon probability for 2N SRC relative to the deuteron", or $\frac{2}{A}(N_{pn(S=1)} + N_{pn(S=0)} + N_{pp} + N_{nn})$. We stress

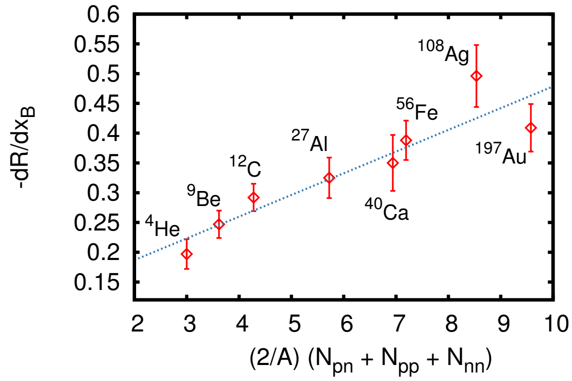

Figure 3. The magnitude of the EMC effect versus our predictions for the "per nucleon probability for 2N SRC relative to the deuteron", $\frac{2}{A}(N_{pn(S=1)}+N_{pn(S=0)}+N_{pp}+N_{nn})$. The data are from the analyses presented in Refs. [16, 21, 22]. The fitted line obeys the equation $-\frac{dR}{dx_B} = (0.11 \pm 0.03) + (0.036 \pm 0.005) \cdot \frac{2}{A}(N_{pn} + N_{pp} + N_{nn})$.

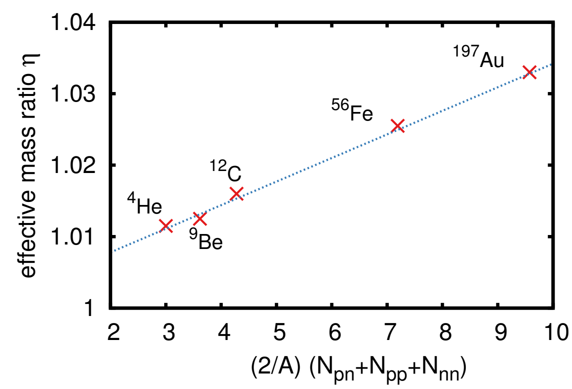

Figure 4. Relation between the effective mass ratio $\eta = \frac{m}{m^*}$ of Ref. [24] and the "per nucleon probability for 2N SRC relative to deuteron". The fitted line obeys the equation $\eta = (1.0012 \pm 0.0006) + (0.0033 \pm 0.0001) \cdot \frac{2}{A}(N_{pn} + N_{pp} + N_{nn})$.

that the numbers which one finds on the x-axis are the results of parameter-free calculations. We consider the "per nucleon probability for 2N SRC relative to the deuteron" as a measure for the magnitude of the nucleon-nucleon SRC in a given nucleus. As can be seen in Fig. 3, there is a nice linear relationship between the quantity which we propose as a per nucleon measure for the magnitude of the SRC and the magnitude of the EMC effect.

In Ref. [23] a formalism to describe the EMC effect was developed by introducing an effective mass. In this formalism the nucleons bound in a nuclei are assumed to have a different effective mass m^* than the free nucleon mass m. A calculated ratio of nuclear to free structure function is fitted to the experimental values of the nucleus to deuteron structure function. The nuclear structure function is a convolution of the free structure function and a distribution function which accounts for Fermi smearing and binding effects. The effective mass remains the only free parameter which is fixed by the fit. The formalism is quite efficient in describing the EMC data. In Fig. 4 we relate the latest value for the ratio of the free nucleon mass to the effective one, $\eta = \frac{m}{m^*}$ [24], to our calculated "per nucleon probability for 2N SRC relative to deuteron". It is obvious that there is a linear relation between the effective mass parameter, used to describe the EMC effect and our "per nucleon probability for 2N SRC relative to the deuteron", which is our estimate for the amount of nucleon-nucleon pairs in a nucleus prone to correlations.

4 Conclusion

We have provided arguments that the mass dependence of the magnitude of the NN correlations can be captured by some approximate principles. Our method is based on the assumption that correlation operators generate the correlated part of the nuclear wave function from that part of the mean-field wave function where two nucleons are spatially "sufficiently close". We have calculated the number of pn, pp and nn ($n_{12} = 0, l_{12} = 0$) SRC-prone pairs and studied their mass and isospin dependence. The A dependence of the magnitude of the pp, nn, and pn SRC can be captured in a in a power-law dependence, A^α with $\alpha = 1.35 \pm 0.03$.

We related the experimentally determined scaling parameter $a_2\,(A/D)$ to our computed "per nucleon probability for a pn SRC relative to deuterium". To connect the computed number of SRC pairs to the measured $a_2\,(A/D)$ corrections are in order. Published experimental data include the radiation and Coulomb corrections. The correction factor stemming from final-state interactions and from the c.m. motion of the correlated pair, however, is far from established. After correcting for the c.m. motion of pairs in a finite nuclei, our model calculations for a_2 are of the right order of magnitude. We predict a rather soft mass dependence which for heavy nuclei, however, is stronger than what the experiments indicate. It remains to be studied whether final-state interactions can account for this additional mass dependent correction factor.

We find a linear relationship between the magnitude of the EMC effect and the computed per nucleon number of SRC-prone pairs. Also other parameters used to describe the EMC effect, like the effective mass parameter, tend to have a linear relationship to our predictions for the per nucleon number of SRC 2N pairs. Those may indicate that the EMC effect is (partly) driven by local nuclear dynamics (fluctuations in the nuclear densities), and that the number of SRC-prone pairs serves as a measure for the magnitude of this effect.

Acknowledgments

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