

Density dependence of single-proton and two-proton knockout reactions under quasifree conditions

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We consider high-energy quasifree single- and two-proton knockout reactions induced by electrons and protons and address the question of what target-nucleus densities can be effectively probed after correcting for nuclear attenuation (initial- and final-state interactions). Our calculations refer to ejected proton kinetic energies of 1.5 GeV, the reactions ($e, e'p$), (γ, pp), and ($p, 2p$), and a carbon target. It is shown that each of the three reactions is characterized by a distinctive sensitivity to the density of the target nucleus. The bulk of the (γ, pp) strength stems from the high-density regions in the deep nuclear interior. Despite the strong attenuation, sizable densities can be probed by ($p, 2p$) reactions provided that the energy resolution allows one to pick nucleons from s orbits. The effective mean densities that can be probed in high-energy ($e, e'p$) reactions is of the order of 30%–50% of the nuclear saturation density.

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Nucleon knockout studies in quasifree kinematics lie in the realm of the most powerful instruments for studying the structure of nuclei. Since the 1960s the $A(e, e'p)$ reaction has provided a wealth of information about the merits and limitations of the nuclear shell model [1]. Quasifree proton scattering from nuclei in the $A(p, 2p)$ reaction has a somewhat longer history [2] and could in principle provide information similar to that of $A(e, e'p)$ reactions. With three protons subject to attenuation effects, in the $A(p, 2p)$ reaction the description of the initial- and final-state interactions (ISIs and FSIs, respectively) is a more challenging issue than in $A(e, e'p)$ reactions. Recent applications of $A(p, 2p)$ reactions involve the analyzing power (A_y) as an instrument for probing medium modifications of hadron properties and the density dependence of the nucleon-nucleon interaction [3,4]. In inverse kinematics [i.e., the $p(A, 2p)A - 1$ reaction] the ($p, 2p$) process offers great opportunities for systematic studies of the density and isospin dependencies of single-particle properties in unstable nuclei [5] at high-energy radioactive beam facilities [6]. Experiments of that type have the potential to study the equation of state for nuclei far from equilibrium. With regard to quasifree $A(e, e'p)$ reactions, recent developments include the search for medium modifications of electromagnetic form factors through double polarization experiments of the type ${}^4\text{He}(\vec{e}, e'\vec{p})$ [7]. Another line of current research is that of the two-nucleon removal reactions $A(e, e'pp)$ and $A(\gamma, pp)$ in selected kinematics. These reactions provide a window into the short-range structure of nuclei [8].

The development of an appropriate reaction theory is essential for reliably extracting physical information from nucleon knockout reactions. For nucleon kinetic energies up to about 1 GeV the distorted wave impulse approximation (DWIA) has achieved considerable success in that it can reproduce a

large amount of measurements fairly well [1]. At sufficiently high nucleon energies the Glauber approach provides a valid alternative to the DWIA framework. In the Glauber approach, the effect of ISIs and FSIs can be computed from knowledge of the elementary proton-proton and proton-neutron differential cross sections and of the density of the target (residual) nucleus. For nucleon momenta exceeding about 1 GeV, the energy dependence of the parameters entering the Glauber calculations is relatively smooth. This results, for example, in measured and computed nuclear $A(e, e'p)$ transparencies that exhibit little energy dependence for nucleon kinetic energies larger than 0.5 GeV [9]. From the theoretical point of view, it allows one to make general statements about the predicted role of nuclear attenuation. The Glauber approach is applicable to a wide range of reactions, including electromagnetic and hadronic probes, with both stable and unstable nuclei [10,11].

We exploit the robustness of the Glauber approach to study the density dependence of quasifree nucleon removal reactions. Indeed, investigations into the medium dependence of nucleon properties and the study of the nuclear structure of unstable nuclei heavily rely on the possibility of effectively probing regions of sufficiently high density in the target nucleus. Nuclear attenuation effects on the impinging and ejected protons can cause the nucleon knockout reactions to effectively probe regions of relatively small density near the surface of the target nucleus. The description of nuclear attenuation brings in a certain degree of model dependence. We stress the importance of making cross-checks over different types of reactions (electromagnetic versus hadronic probes) and linking single-nucleon to two-nucleon knockout reactions. Here, we report on a study of the effective nuclear density that can be probed in reactions that have one nucleon [$A(e, e'p)$], two nucleons [$A(\gamma, pp)$], and three nucleons [$A(p, 2p)$] subject to nuclear attenuation effects.

In Ref. [12] a relativistic extension of the Glauber method, called the relativistic multiple-scattering Glauber approximation (RMSGGA), was introduced. In line with the assumptions

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of a typical relativistic DWIA model, the RMSGA model uses a relativistic mean field to describe the target and residual nucleus in combination with the impulse approximation for the interaction Hamiltonian.

In a factorized approach, the differential cross sections for the single-nucleon removal reactions considered here [i.e., $A(p, 2p)$ and $A(e, e'p)$] are proportional to the distorted momentum distributions $\rho_{nk}^D(\vec{p}_m)$:

$$\begin{aligned} \rho_{nk}^D(\vec{p}_m) &= \sum_{s,m} \left| \int d\vec{r} \frac{e^{-i\vec{p}_m \cdot \vec{r}}}{(2\pi)^3} \bar{u}(\vec{p}_m, s) \hat{S}_{\text{RMSGA}}^\dagger(\vec{r}) \phi_{nkm}(\vec{r}) \right|^2 \\ &= \sum_{s,m} (\phi_{nkm}^D)^\dagger \phi_{nkm}^D \\ &= \frac{1}{2} \int dr \int d\theta \left[\sum_{s,m} ((D(r, \theta))^\dagger \phi_{nkm}^D \right. \\ &\quad \left. + D(r, \theta) (\phi_{nkm}^D)^\dagger) \right] \\ &\equiv \int dr \int d\theta \delta(r, \theta), \end{aligned} \quad (1)$$

where the quantum numbers (nkm) determine the orbit of the struck nucleon, $\phi_{nkm}(\vec{r})$ is the corresponding relativistic single-particle wave function, and $u(\vec{p}, s)$ is a four-component free-particle spinor. The missing momentum \vec{p}_m is determined by the difference between the asymptotic three-momentum of the ejected nucleon \vec{p} and the three-momentum transfer \vec{q} . We define the z -axis along \vec{q} and the xz -plane as the reaction plane. The function $\delta(r, \theta)$ defined in Eq. (1) encodes the

contribution from an infinitesimal interval around r and θ to a single-nucleon removal cross section [3]. The function $D(r, \theta)$ introduced in Eq. (1) reads

$$D(r, \theta) = \int d\phi r^2 \sin \theta \frac{e^{-i\vec{p}_m \cdot \vec{r}}}{(2\pi)^3} \bar{u}(\vec{p}_m, s) \hat{S}_{\text{RMSGA}}^\dagger(\vec{r}) \phi_{nkm}(\vec{r}). \quad (2)$$

The Glauber phase operator $\hat{S}_{\text{RMSGA}}^\dagger(\vec{r})$ encodes the combined effect of the initial- and final-state interactions. By retaining only central interactions, which is a fair approximation at the kinetic energies considered here, it can be considered as a scalar operator. For the $A(p, 2p)$ reaction, \hat{S}_{RMSGA} becomes a multidimensional convolution over the squared wave functions of the spectator nucleons times the profile functions for the impinging proton and two ejected protons [13]. For the $A(e, e'p)$ case the convolution involves the squared wave functions of the spectator nucleons times the profile function for the ejected nucleon [12].

We now wish to formulate the analog of Eq. (1) for two-nucleon knockout reactions of the type $A(e, e'pp)$ and $A(\gamma, pp)$. The asymptotic three-momenta of the two ejected protons are defined as \vec{p}_1 and \vec{p}_2 . We introduce relative $\vec{p} = \frac{\vec{p}_1 - \vec{p}_2}{2}$ and center-of-mass $\vec{P} = \vec{p}_1 + \vec{p}_2$ momenta of the ejected pair. Factorizing the $A(\gamma, pp)$ and $A(e, e'pp)$ cross sections requires the assumption that the sudden emission of two correlated protons only occurs when they reside in a relative S state [14]. This is a reasonable approximation because investigations of the $^{16}\text{O}(e, e'pp)$ reaction at the electron accelerators in Mainz [15] and Amsterdam [16] have shown that proton pairs are exclusively subject to short-range correlations when they reside in a relative S state under

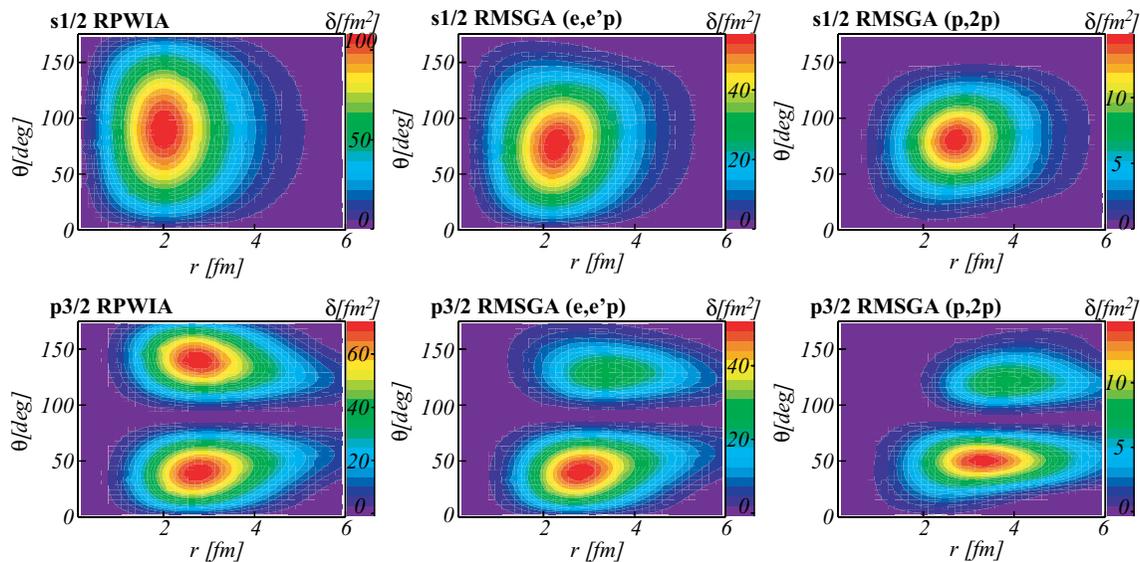


FIG. 1. (Color online) The function $\delta(r, \theta)$ for the $^{12}\text{C}(e, e'p)$ and $^{12}\text{C}(p, 2p)$ reactions. For both types of reaction we consider an energy transfer of 1.5 GeV and a three-momentum transfer \vec{q} that is tuned to probe the maximum of the momentum distribution (i.e., $p_m = 0$ MeV for knockout from the $s_{1/2}$ orbit and $p_m = 115$ MeV for removal from the $p_{3/2}$ orbit). For the $(e, e'p)$ results the proton is detected along the direction of the momentum transfer. For the $(p, 2p)$ reaction the incoming proton has a kinetic energy of about 3 GeV and the two ejected protons have a kinetic energy of 1.5 GeV. They are detected at an angle of about 32° but on opposite sides of the incoming beam. For the sake of reference, the proton root-mean-square radius in ^{12}C as determined from elastic electron scattering is $r_{\text{rms}} = \langle r^2 \rangle^{1/2} = 2.464 \pm 0.012$ fm [17].

conditions corresponding with relatively small center-of-mass momenta P . In other words, the distinctive feature of the two correlated protons is that they are very close and moving back to back. With the relative S -state assumption, the differential cross section is proportional to the distorted momentum distribution corresponding with the center-of-mass motion of the pair,

$$\begin{aligned} \rho_{n_1\kappa_1, n_2\kappa_2}^D(\vec{P}) &= \sum_{s_1, s_2, m_1, m_2} \left| \int d\vec{R} \frac{e^{-i\vec{P}_m \cdot \vec{R}}}{(2\pi)^3} \bar{u} \left(\vec{p} + \frac{\vec{P}}{2}, s_1 \right) \right. \\ &\quad \times \phi_{n_1\kappa_1, m_1}(\vec{R}) \bar{u} \left(-\vec{p} + \frac{\vec{P}}{2}, s_2 \right) \\ &\quad \left. \times \phi_{n_2\kappa_2, m_2}(\vec{R}) \widehat{S}_{\text{RMSGGA}}^\dagger(\vec{R}) \right|^2 \\ &\equiv \int dR \int d\theta \delta(R, \theta), \end{aligned} \quad (3)$$

where $\vec{P}_m = \vec{p}_1 + \vec{p}_2 - \vec{q}$ is the pair's missing momentum in the quasifree approximation. It can be interpreted as the center-of-mass momentum of the correlated proton pair that absorbs the photon. The quantum numbers $(n_1\kappa_1, n_2\kappa_2)$ determine the orbits of the two correlated nucleons.

We now present the results of the numerical calculations for $\delta(r, \theta)$ and $\delta(R, \theta)$. A relativistic single-particle model was used for ^{12}C with parameters adjusted to describe the ground-state properties. The profile functions entering the $\widehat{S}_{\text{RMSGGA}}$ operator have three parameters that have been determined from the database of proton-proton and proton-neutron cross sections [12]. In Fig. 1 we display the function $\delta(r, \theta)$ defined in the Eq. (1) for proton knockout from the $s_{1/2}$ and $p_{3/2}$ orbits from a ^{12}C target. We compare the $(p, 2p)$ with the $(e, e'p)$ results for an energy transfer of 1.5 GeV and conditions probing the maximum of the undisturbed momentum distribution $\rho_{nk}(\vec{p})$. The latter can be obtained by setting $\widehat{S}_{\text{RMSGGA}} = 1$ in Eq. (1). In the considered kinematics, it is clear that, in the absence of nuclear attenuation ($\widehat{S}_{\text{RMSGGA}} = 1$), the upper hemisphere ($0^\circ \leq \theta \leq 90^\circ$) and lower hemisphere ($90^\circ \leq \theta \leq 180^\circ$) of the target nucleus equally contribute to $\delta(r, \theta)$ and the measured signal. Moreover, $\delta(r, \theta)$ becomes equal for $(e, e'p)$ and $(p, 2p)$ reactions. We refer to this situation as the relativistic plane-wave impulse approximation (RPWIA). Because the ISI and FSI have the strongest impact at the highest nuclear densities, the RMSGGA predictions for $\delta(r, \theta)$ are shifted to larger values of r in comparison with the RPWIA ones. In addition, the contribution from the upper and lower hemispheres becomes asymmetric when attenuation is considered. Indeed, the nuclear hemisphere closest to the proton detector provides the strongest contribution to the detected signal. The stronger the effect of attenuation the more $\delta(r, \theta)$ experiences a shifts in r , the larger the induced asymmetries between the upper and lower hemisphere become, and the stronger the reduction becomes. Obviously, the asymmetry, shift, and reduction occur for $\delta(r, \theta)$ in $(e, e'p)$ and $(p, 2p)$ reactions. All three effects, however, are far more pronounced for the $(p, 2p)$

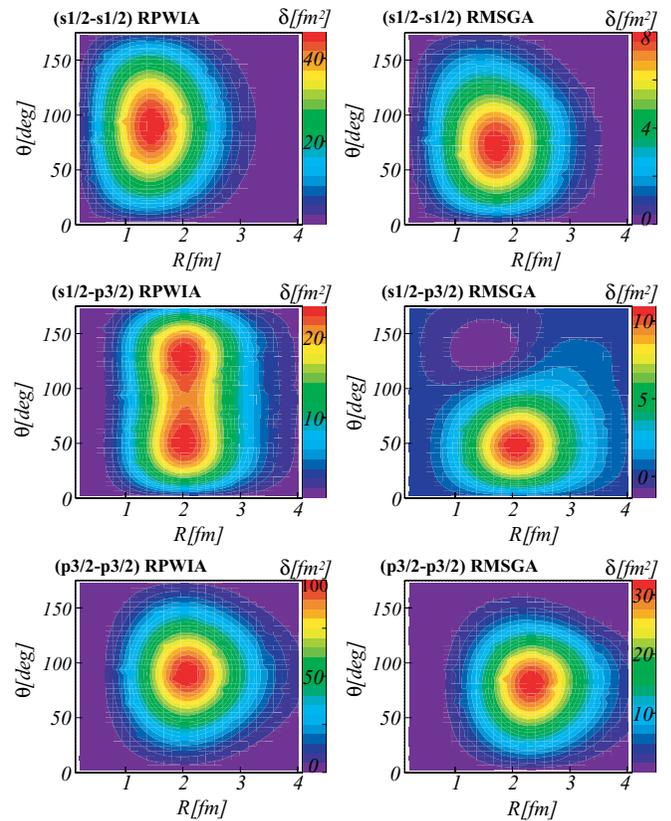


FIG. 2. (Color online) The function $\delta(R, \theta)$ for the exclusive $^{12}\text{C}(\gamma, pp)$ cross section. In all situations we consider an energy transfer of 3 GeV and a three-momentum transfer \vec{q} that is tuned to probe the maximum of the momentum distribution $\rho_{n_1\kappa_1, n_2\kappa_2}(\vec{P})$ [i.e., $P = 0$ MeV for knockout from the $(s_{1/2}-s_{1/2})$ and $(p_{3/2}-p_{3/2})$ orbits and $P = 160$ MeV for removal from the $(s_{1/2}-p_{3/2})$ orbits]. We consider coplanar and symmetric kinematics (i.e., the two escaping protons have the same energy and polar angle θ_{pq} , but they escape from the opposite side of \vec{q}).

reaction than for the corresponding $\delta(r, \theta)$ in the $(e, e'p)$ reaction.

In Fig. 2 we display for the ^{12}C target the function $\delta(R, \theta)$ defined in Eq. (3) for two-proton knockout from the $(p_{3/2}-p_{3/2})$, $(s_{1/2}-s_{1/2})$, and $(s_{1/2}-p_{3/2})$ orbits. By comparing Figs. 1 and 2 it is clear that two-proton removal at high energies really succeeds in probing the high-density regions of the target nucleus. (Note the different range in the radial coordinate r for Figs. 1 and 2.) The attenuation mechanisms induce shifts to the surface but the bulk of the measured strength can be clearly attributed to high-density regions in the target nucleus.

To quantify the average densities that the various reactions can probe, we introduce [3,4]

$$\bar{\rho} = \frac{\int dr d\theta \rho(\vec{r}) \delta(r, \theta)}{\int dr d\theta \delta(r, \theta)}, \quad (4)$$

where $\rho(\vec{r})$ is the density of the target nucleus and $\delta(r, \theta)$ is the function defined in Eq. (1) (for single-proton knockout) and Eq. (3) (for two-proton knockout). Table I lists a systematic

TABLE I. The average effective density $\bar{\rho}$ and rms radii probed in the various reactions.

Reaction	Orbits	$\bar{\rho}$ (RPWIA) (fm ⁻³)	$\bar{\rho}$ (RMSGGA) (fm ⁻³)	r_{rms} (RPWIA) (fm)	r_{rms} (RMSGGA) (fm)
(<i>p</i> , 2 <i>p</i>)	<i>s</i> _{1/2}	0.100	0.055	2.312	2.967
(<i>p</i> , 2 <i>p</i>)	<i>p</i> _{3/2}	0.050	0.025	3.113	3.708
(<i>e</i> , <i>e'</i> <i>p</i>)	<i>s</i> _{1/2}	0.100	0.086	2.312	2.512
(<i>e</i> , <i>e'</i> <i>p</i>)	<i>p</i> _{3/2}	0.050	0.038	3.113	3.353
(γ , <i>pp</i>)	(<i>s</i> _{1/2} - <i>s</i> _{1/2})	0.150	0.135	1.629	1.816
(γ , <i>pp</i>)	(<i>p</i> _{3/2} - <i>p</i> _{3/2})	0.095	0.075	2.318	2.567
(γ , <i>pp</i>)	(<i>s</i> _{1/2} - <i>p</i> _{3/2})	0.115	0.098	2.064	2.253

comparison of the computed values of $\bar{\rho}$ and r_{rms} for the different reactions. The average density probed in the two-proton removal reaction from the (*s*_{1/2}-*s*_{1/2}) orbits approaches the nuclear saturation density of $\rho_0 = 0.17 \text{ fm}^{-3}$. We wish to stress the strong dependence on the nuclear orbit. Despite the strong attenuation, the (*p*, 2*p*) reaction from the *s*_{1/2} orbit can effectively probe higher densities than the (*e*, *e'* *p*) reaction from the valence *p*_{3/2} shell. For the (*p*, 2*p*) reaction with knockout from the *s*_{1/2} orbit the predicted effective mean density from the RMSGGA calculations is $\bar{\rho} \approx 0.33\rho_0$. This number is almost identical to the DWIA results of Ref. [4] for ¹²C(*p*, 2*p*) for 1-GeV incoming protons. The fractional reduction in $\bar{\rho}$ in going from RPWIA to RMSGGA is of the order 50%–55% in (*p*, 2*p*) reactions, 75%–85% in (*e*, *e'* *p*) reactions, and 80%–90% in (γ , *pp*) reactions. The corresponding fractional increase in r_{rms} is of the order of 10% for (γ , *pp*) and (*e*, *e'* *p*) reactions, whereas it reaches 20%–30% for (*p*, 2*p*) reactions. It is striking that despite one additional nucleon in the final state, on average the fractional reduction in $\bar{\rho}$ is less for (γ , *pp*) than for (*e*, *e'* *p*) reactions. This reflects the peculiar radial dependence of the density profiles for the different reactions and the fact that, for an

identical orbit, r_{rms} is substantially smaller for (γ , *pp*) than for (*e*, *e'* *p*) reactions.

In summary, we have used a relativistic framework to make a comparative and consistent study of the effective nuclear densities that can be probed in (*p*, 2*p*), (*e*, *e'* *p*), and (γ , *pp*) reactions. As a representative example we have selected a carbon target and ejected proton kinetic energies of 1.5 GeV. We consider the results as representative for light nuclei and sufficiently high kinetic energies. The conclusions drawn in this work are of importance for ongoing and planned research of nuclear effects at small distance scales. The (*e*, *e'* *p*) reaction has the potential to probe reasonable densities. Of all reactions considered here, the (γ , *pp*) reaction is the one that can get closest to the deep nuclear interior. The (*p*, 2*p*) reaction is subject to large attenuation, but a high-resolution experiment picking protons from *s* orbits, for example, can probe densities that are of the order of 30% of ρ_0 .

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