How to probe the infrared quark mass with parity quartets in the high baryon spectrum

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Abstract

By current theoretical understanding, spontaneous chiral symmetry breaking enhances the quark masses in the infrared, and thus generates most of the visible mass in our universe, i. e. the mass of the nucleons, while simultaneously removing chiral symmetry from the lowest states of the light hadron spectrum.

We first show that three-quark states naturally group into quartets (with two states of each parity), split into two parity doublets, all splittings decreasing high in the spectrum. We then present a first theoretical computation of the spectrum of high-J excited baryons with a chiral invariant quark model.

We propose that a measurement of masses of high-partial wave Δ resonances with an accuracy of 50 MeV should be sufficient to unambiguously establish the approximate degeneracy, and learn how to probe the running quark mass in the mid-infrared power-law regime, thanks to an ultrarelativistic expansion of the quark spinors. Such precision, if challenging, can be reached thanks to experimental progress at ELSA, Jefferson Lab and other facilities, and to the large effort of the lattice QCD community to compute the baryon spectrum.

Key words: Chiral symmetry restoration, parity doubling, baryon quartets, running quark mass, QCD spectrum.
PACS:

1 Introduction

Quantum Chromodynamics (QCD) is a theory of the strong nuclear force that has been thoroughly tested in high-energy physics, through hadron jets, Drell-Yan processes, electron-positron annihilation, deep inelastic scattering and several other experiments. Low-energy QCD manifests chiral symmetry breaking (χSB), that leads to several established hadron "low energy" theorems. χSB also produces the enhancement of the quark masses in the infrared, generating most of the visible mass in our universe. For the same token, chiral symmetry is removed from the ground states of light hadron spectrum (so low-lying hadrons do not appear in parity doublets).

However, so far, the middle and high range of the spectrum of light quark hadrons, extending between about 1 GeV and 3 GeV, has not been accessible to any known form of perturbation theory, effective or fundamental. Insensitivity to chiral symmetry breaking, recently stressed by Glozman [1,2,3,4,5,6], and restrospectively present in the excited hadronic spectra computed in chiral invariant quark models [7,8] has led many hadron physicists to accept that spontaneous chiral symmetry breaking, the salient feature of the low hadron spectrum, actually is of lesser importance for excited resonances, to the extent that "high enough" in the spectrum, chiral symmetry is asymptotically realized in the Wigner mode, by which hadrons must present themselves in degenerate chiral multiplets [9]. We have realized that a formalization of this statement is that the ratio of the mass to the momentum of the quark $\langle \frac{m}{k} \rangle$ provides a new perturbative parameter to study some aspects of the middle and high range of the hadron spectrum.

We also provide the theoretical background to understand parity doubling in baryons. So far quark-based calculations implementing chiral symmetry breaking have only been applied to meson (quark-antiquark) resonances. We now show that three-quark states naturally group into quartets with two states of each parity. Diagonalizing the chiral charge expressed in terms of quarks we find that the quartet is split into two parity doublets, all splittings decreasing towards the ultraviolet high in the spectrum.

Finally, we compute for the first time in a chiral invariant quark model, an excited baryon spectrum. For reasons of simplicity and to minimize the impact of molecular meson-nucleon configurations [10], we study the family of maximum-spin excitations Δ^* of the Delta baryon, the leading Regge trajectory of the Δ baryon spectrum. We estimate the experimental accuracy needed to establish Chiral Symmetry Restoration in the high spectrum, to help quantify what "high enough" in the spectrum means, and assist experimental planning.

Experimentally, the Δ^* excitations of the nucleon are copiously pion-, photo- or electroproduced. The Δ^{++} are especially easy to track by their decay into a proton p^+ and a pion π^+. In all, we advocate that a measurement of masses of high-partial wave Δ resonances with an accuracy of 50 MeV should be sufficient to unambiguously establish the approximate degeneracy, and test the concept of running quark mass in the infrared.

2 Infrared Enhancement of the Quark Mass and the Utraviolet Insenvitivity to Chiral Symmetry Breaking

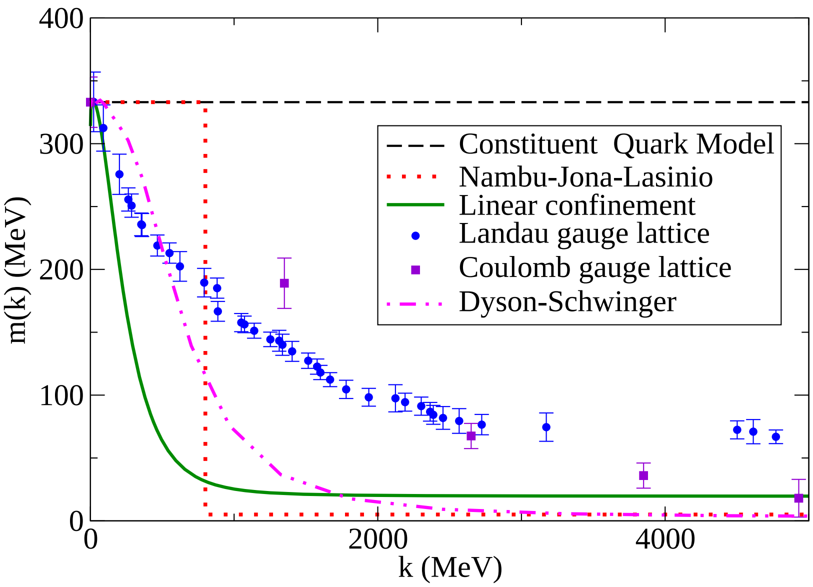

Fig. 1. Infrared enhancement of the light quark mass, generated when spontaneous chiral symmetry breaking occurs. Shown are quark masses in the main approaches to QCD, all multiplied by an arbitrary factor to match them at zero momentum.

The quark mass is supposed to run from some high, perturbative scale, where $m \simeq 1 - 5\ MeV$, to a constituent mass of circa 300 MeV, a whole two orders of magnitude. Chiral perturbation theory and lattice QCD assist quark mass extraction (above all, ratios between different flavors) [11], and both the lattice and the Dyson-Schwinger approaches to QCD show its running towards the infrared, but little has been said about this experimentally. In Fig. 1, we show this crucial dynamical infrared enhecement of the quark mass, generated when spontaneous χSB occurs. All masses are multiplied by an arbitrary factor to match them at zero momentum. The constituent quark model, for simplicity, considers a constant quark mass [12]. The oldest model of chiral symmetry breaking, the (quark) Nambu and Jona-Lasinio model presents a step-function infrared mass enhancement [13]. In the last two decades, this NJL model has been extended to include confinement, resulting in the chirally invariant quark model. With a linear confining potential, it is known that the transition from the dynamically generated infrared mass to the ultraviolet mass follows a power-law $m \to \frac{C}{k^4}$, with C a constant [14]. Another power-law result, although for Euclidean momentum, is found when the mass is generated with simplified Schwinger-Dyson equations in the Landau gauge [15], and when the mass is computed in lattice QCD in the Landau gauge as adapted from Bowman et al. [16,17].

The Dirac spinors $U_{\kappa\lambda}$ and $V_{-\kappa\lambda}$ rule the quark-quark and quark-antiquark interaction. Widely used is the heavy quark limit, both in quark model or lattice NRQCD computations, where spin-tensor potentials are successfully derived with the spinor expansion in orders of $k/m(k)$. For light quarks the opposite, ultrarelativistic (large-momentum) limit is relevant. Spinors are then conveniently expanded in the inverse ratio $m(k)/k$, or, with $E(k) = \sqrt{k^2 + m(k)^2}$,

$$U_{\kappa\lambda} = \frac{1}{\sqrt{2E(k)}} \begin{bmatrix} \sqrt{E(k)+m(k)}\chi_\lambda \\ \sqrt{E(k)-m(k)}\vec{\sigma}\cdot\hat{\kappa}\chi_\lambda \end{bmatrix} \underset{k\to\infty}{\longrightarrow} \frac{1}{\sqrt{2}} \begin{bmatrix} \chi_\lambda \\ \vec{\sigma}\cdot\hat{\kappa}\chi_\lambda \end{bmatrix} + \frac{1}{2\sqrt{2}}\frac{m(k)}{k} \begin{bmatrix} \chi_\lambda \\ -\vec{\sigma}\cdot\hat{\kappa}\chi_\lambda \end{bmatrix} \quad (1)$$

having kept the leading chiral invariant term, and a next order chiral symmetry breaking $\frac{m(k)}{k}$ term. Non-chiral, spin-dependent potentials in the quark-quark interaction originate from the second term in the expansion eq. (1).

Our approach entails an expansion of H^{QCD} in the weak sense, that is, not of the Hamiltonian operator itself, but a restriction thereof to the Hilbert space of highly excited resonances, where average quark momentum is large, or, in obvious notation,

$$\langle\psi_m|H^{QCD}|\psi_n\rangle \simeq \langle\psi_m|H_\chi^{QCD}|\psi_n\rangle + \langle\psi_m|\frac{m(k)}{k}H_\chi^{QCD\,\prime}|\psi_n\rangle + \dots \quad (2)$$

3 Baryon quartets

The approximate degeneracy into chiral quartets follows from invariance under chiral transformations. These act on the classical quark fields as [18] $\psi \to e^{i\theta\gamma_5}\psi$. Since the classical QCD Lagrangian contains no chiral-symmetry breaking interactions of the form $gA^{a\mu}\bar{\psi}\partial_\mu\psi$, and only small current quark mass terms $m_0\bar{\psi}\psi$, there is an (approximately) conserved chiral charge at the classical level $Q_5^a = \int d\mathbf{x}\psi^\dagger(x)\gamma_5\frac{\tau^a}{2}\psi(x)$ due to Noether's theorem. Upon quantizing, $[Q_5^a, H] = 0$. However Chiral Symmetry is spontaneously broken by the ground state, $Q_5^a|0\rangle \neq 0$ leading to the appearance of a large quark mass in the quark propagator, $m(k)$, to pseudo-Goldstone bosons (the pseudoscalar meson octet of $\pi,\ K,\ \eta$), and to the loss of the chiral degeneracy in the groundstate baryons.

Substituting the spinors, and translating the quark and antiquark operators in terms of Bogoliubov-rotated quark and antiquark normal modes B, D [19], the chiral charge takes the form

$$Q_a^5 = \int \frac{d^3k}{(2\pi)^3} \sum_{\lambda\lambda' f f' c} \left(\frac{\tau^a}{2}\right)_{ff'} \tag{3}$$

$$\left(\frac{k}{\sqrt{k^2+m^2(k)}} (\sigma\cdot\hat{\mathbf{k}})_{\lambda\lambda'} \left(B^\dagger_{k\lambda fc} B_{k\lambda' f' c} + D^\dagger_{-k\lambda' f' c} D_{-k\lambda fc} \right) + \right.$$

$$\left. \frac{m(k)}{\sqrt{k^2+m^2(k)}} (i\sigma_2)_{\lambda\lambda'} \left(B^\dagger_{k\lambda fc} D^\dagger_{-k\lambda' f' c} + B_{k\lambda' f' c} D_{-k\lambda fc} \right) \right) .$$

In the presence of Spontaneous Chiral Symmetry Breaking, $m(k) \neq 0$, and the two terms in the third line are responsible for the non-linear realization of chiral symmetry in the spectrum as they create/destroy a pion.

But when the average momentum of any state on which this operator acts is large, it is the second line that dominates, and chiral symmetry is realized linearly (as it only displays quark counting operators with a parity-changing spin flip).

We now examine the effect of a chiral transformation on a three-quark variational wavefunction $|N\rangle = F_{ijk} B^\dagger_i B^\dagger_j B^\dagger_k |0\rangle$. If a resonance is high enough in the spectrum, the quarks have a momentum distribution peaked higher than the infrared momenta where the mass is dynamically enhanced, and only the second line of Eq.(3) is active. $Q_5|N\rangle$ contains also three quarks, but one of them is spin-rotated from $B_{k\lambda}$ to $\sigma\cdot\hat{k}_{\lambda\lambda'} B_{k\lambda'}$. Successive application of the chiral charge spin-rotates further quarks, changing each time the parity of the total wavefunction. However the sequence of states is closed since $\sigma\cdot\hat{k}\sigma\cdot\hat{k} = \mathbb{I}$. In fact, starting with an arbitrary wavefunction with parity P, one generates a quartet, where in the following we drop the isospin index,

$$\begin{aligned}
|N_0^P\rangle &= \sum F^P_{ijk} B^\dagger_i B^\dagger_j B^\dagger_k |\Omega\rangle && (4)\\
|N_1^{-P}\rangle &= \frac{1}{3}\sum F^P_{ijk} \left(\left(\sigma\cdot\hat{\mathbf{k}}B^\dagger\right)_i B^\dagger_j B^\dagger_k + B^\dagger_i \left(\sigma\cdot\hat{\mathbf{k}}B^\dagger\right)_j B^\dagger_k + B^\dagger_i B^\dagger_j \left(\sigma\cdot\hat{\mathbf{k}}B^\dagger\right)_k \right) |\Omega\rangle \\
|N_2^P\rangle &= \frac{1}{3}\sum F^P_{ijk} \Big(\left(\sigma\cdot\hat{\mathbf{k}}B^\dagger\right)_i \left(\sigma\cdot\hat{\mathbf{k}}B^\dagger\right)_j B^\dagger_k + B^\dagger_i \left(\sigma\cdot\hat{\mathbf{k}}B^\dagger\right)_j \left(\sigma\cdot\hat{\mathbf{k}}B^\dagger\right)_k \\
&\qquad + \left(\sigma\cdot\hat{\mathbf{k}}B^\dagger\right)_i B^\dagger_j \left(\sigma\cdot\hat{\mathbf{k}}B^\dagger\right)_k \Big) |\Omega\rangle \\
|N_3^{-P}\rangle &= \sum F^P_{ijk} \left(\sigma\cdot\hat{\mathbf{k}}B^\dagger\right)_i \left(\sigma\cdot\hat{\mathbf{k}}B^\dagger\right)_j \left(\sigma\cdot\hat{\mathbf{k}}B^\dagger\right)_k |\Omega\rangle
\end{aligned}$$

that is the natural basis to discuss chiral symmetry restoration in baryons, through wavefunctions that are linear combinations $|N\rangle = \sum c_a |N_a\rangle$.

Because the Hamiltonian and the chiral charge commute, they can be diagonalized simultaneously. The representation of the chiral charge in the (non-orthonormal) quartet coordinates can be broken in two blocks since Q_5 changes parity, if one uses the square charge

$$Q_5^2 \begin{pmatrix} c_0 \\ c_2 \end{pmatrix} = \begin{bmatrix} 3 & 2 \\ 6 & 7 \end{bmatrix} \begin{pmatrix} c_0 \\ c_2 \end{pmatrix} . \tag{5}$$

Immediately one sees that the two linear combinations $N_0 - N_2$ and $N_0 + 3N_2$ diagonalize the square chiral charge in the positive parity sector, with $N_1 - N_3$ and $3N_1 + N_3$ doing so in the negative parity.

The quartet then separates into two doublets connected by the chiral charge

$$\begin{array}{lcl} Q_5(N_0 - N_2) = N_1 - N_3 \ , & & Q_5(N_1 - N_3) = N_0 - N_2 \\ Q_5(N_0 + 3N_2) = 3(3N_1 + N_3) \ , & & Q_5(3N_1 + N_3) = 3(N_0 + 3N_2) \end{array} \tag{6}$$

Since the quartet can be divided into two two-dimensional irreducible representations of the chiral group (with different eigenvalues of Q_5^2, 1 and 9 respectively), the masses of the two doublets may also be different, and the interdoublet splitting becomes a dynamical question (we will argue shortly that it is small for highly excited baryons). However, the splitting within the doublet *must vanish* asymptotically. Even for fixed (not running) quark mass, when the typical momenta are high enough $\langle k \rangle >> m$ in the kinetic energy, the effects of the quark mass are negligible. Parity doubling then comes down to whether the interaction terms are also chiral symmetry violating or not.

Finally let us remark that, while they are a suboptimal variational basis for low-lying states, the quartet states coincide with the H-eigenstates high in the spectrum. We use both the quartet basis and the fixed L variational basis in our calculations to illustrate the physical principles.

4 Experimental access to the infrared quark mass running

We now exploit the smallness of the $|M^+ - M^-|$ mass difference with increasing angular momentum to guide experiment in obtaining information about the running quark mass. For this we will establish how the j-scaling of this quantity is related to the k-scaling of the running quark mass.
The $M^\pm$ in our proposed study are the masses of the ground state quartets of the baryon spectrum, with parity $\pm$ and in the limit of large total angular momentum j (large when compared with $s = 3/2$). Naturally, the two approximately degenerate masses M^+ and M^- are both part of the same

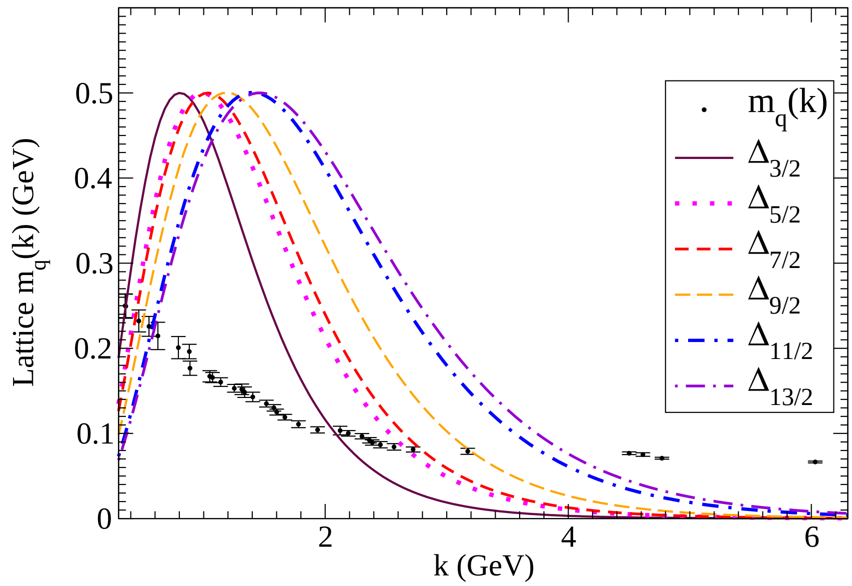

Fig. 2. Typical momentum distributions of increasingly excited $\Delta_{3/2}$, $\Delta_{5/2}$, $\Delta_{7/2}$, $\Delta_{9/2}$, $\Delta_{11/2}$, $\Delta_{13/2}$ resonances overlap less and less with the dynamically generated infrared quark mass. (Illustrative variational wavefunctions for a linear potential with string tension $\sigma = 0.135\ GeV^2$, not normalized for visibility).

leading linear Regge trajectory, phenomenologically fixing their j-scaling

$$j = \alpha_0 + \alpha M^{\pm 2} \underset{j\to\infty}{\longrightarrow} \alpha M^{\pm 2} \ . \tag{7}$$

The parity of the ground state alternates between + an − as the angular momentum steps up by one. Large j is equivalent to large quark orbital angular momentum l since the spin is finite, and also to a large average linear momentum $\langle k \rangle$. This is illustrated in Fig 2 where we show how the overlap with the running quark mass stops perceiving χSB for high-lying states. The relativistic version of the virial theorem [20] states that the kinetic energy is a fixed part of the total energy and thus

$$\langle k \rangle \to c_2\, M^{\pm} \to \frac{c_2}{\sqrt{\alpha}} \sqrt{j} \tag{8}$$

(where c_2 is a constant, for instance for a linear potential, a relativistic kinetic energy, and 3 quarks in a baryon, $c_2 = 1/6$).

The first term in Eq. (2) cancels out in the difference $|M^+ - M^-| << M^{\pm}$ (while each of the masses $M^{\pm}$ is dominated by the chiral invariant term, the mass difference stems from the dynamically generated quark mass) thus exposing the second term in eq.(2), proportional to $\langle \frac{m(k)}{k} \rangle$, viz.

$$|M^+ - M^-| \to \langle \frac{m(k)}{k} H_\chi^{QCD\ \prime} \rangle \to c_3 \frac{m(\langle k \rangle)}{\langle k \rangle} \langle H_\chi^{QCD\ \prime} \rangle \tag{9}$$

(the factorization is allowed by the mean value theorem at the price of an unknown constant that we do not attempt to determine here). This equation is analogous to the renowned Gell-Mann-Oakes-Renner relation

$$M_\pi^2 = -m_q \frac{\langle \bar{\psi}\psi \rangle}{f_\pi^2} \tag{10}$$

but active when chiral symmetry is realized linearly, as in the high-baryon excitations we examine.

$H_\chi^{QCD\,'}$ contains the products of the $\sigma \cdot \hat{\mathbf{k}}$ present in the spinors and thus includes spin-spin, spin-orbit and tensor potentials together with spin independent terms. To obtain its j-scaling we need to examine matrix elements in the limit of large j, equivalent to large $\langle k \rangle$ or large $M^\pm$. We separately consider its angular and radial dependences $\langle H_\chi^{QCD\,'} \rangle \; \alpha \; \langle H_\chi^{QCD\,'} \rangle_{\rm angular} \times \langle H_\chi^{QCD\,'} \rangle_{\rm radial}$. The angular matrix element generally includes a spin-orbit term, that leads its j-counting

$$\langle H_\chi^{QCD\,'} \rangle_{\rm angular} \to j \ . \tag{11}$$

For the radial part we have to notice that in the large momentum limit there is only one scale in the hamiltonian, Λ_{QCD} or equivalently the string tension σ. Thus the highest j-power is provided by the centrifugal barrier, dominating the radial eigenvalue equation

$$l(l+1)\langle H_\chi^{QCD\,'} \rangle_{\rm radial} \to j^2 \langle H_\chi^{QCD\,'} \rangle_{\rm radial} \to c_4 M^\pm \tag{12}$$

and thus $\langle H_\chi^{QCD\,'} \rangle_{\rm radial} \propto j^{-2} M^\pm$. Combining with eq. (11) we get [1]

$$\langle H_\chi^{QCD\,'} \rangle \to c_5 \, M^\pm j^{-1} \to \frac{c_5}{\sqrt{\alpha}} \sqrt{\frac{1}{j}} \ . \tag{13}$$

The result of the j-scaling analysis reads then

$$|M^+ - M^-| \to \frac{c_3 \, m(\langle k \rangle)}{\langle k \rangle} \times c_5 \, M^\pm \times j^{-1} = \frac{c_3 c_5}{c_2} m(\langle k \rangle) \; j^{-1} \ . \tag{14}$$

This equation links the infrared enhancement of the quark mass to baryon spectroscopy in a usable way. An experimental extraction proceeds by just fitting the exponent of the j-scaling for the splitting $|M^+ - M^-| \propto j^{-i}$. Then, in view of eq. (8), one obtains either of

[1] As a corollary, note that a spin-independent potential scales like $\frac{1}{j^{3/2}}$.

$$m(\Lambda \times \sqrt{j}) \propto j^{-i+1} \tag{15}$$
$$m(k) \propto k^{-2i+2} \ . \tag{16}$$

The same exponent i that appears in this last equation can be obtained from the fit to the $|M^+ - M^-|$ with increasing j! [2]

5 Example model calculation of the quartet splittings

Our next contribution is to present for the first time a competitive chirally-invariant quark model computation of the parity doubling in the excited baryon spectrum, shown in figure 3. The model we employ is inspired in Coulomb-gauge QCD (essentially dropping the Faddeev-Popov operator and replacing the Coulomb kernel by its vacuum expectation value known to grow linearly with distance) and can be seen as a field theory upgrade of the Cornell potential model.

The Hamiltonian reads

$$H = -g_s \int d\mathbf{x} \Psi^\dagger(x) \alpha \cdot \mathbf{A}(x) \Psi(x) + Tr \int d\mathbf{x} (\mathbf{E} \cdot \mathbf{E} + \mathbf{B} \cdot \mathbf{B})$$
$$+ \int d\mathbf{x} \Psi_q^\dagger(\mathbf{x}) (-i\alpha \cdot \nabla + \beta m_0) \Psi_q(\mathbf{x}) - \frac{1}{2} \int d\mathbf{x} d\mathbf{y} \rho^a(\mathbf{x}) V_L(|\mathbf{x} - \mathbf{y}|) \rho^a(\mathbf{y})$$

with a strong kernel containing a linear potential V_L, with string tension $\sigma = 0.135\ GeV^2$, coupled to the color charge density $\rho^a(\boldsymbol{x}) = \Psi^\dagger(\boldsymbol{x}) T^a \Psi(\boldsymbol{x}) + f^{abc} \mathbf{A}^b(\boldsymbol{x}) \cdot \mathbf{\Pi}^c(\boldsymbol{x})$. We solve the BCS gap equation to spontaneously break chiral symmetry and employ both the so calculated $m(k)$ quark mass as well as a lattice computation in Landau gauge [17] and in Coulomb gauge known to us[21]. The model has the same chiral structure of QCD[7], satisfying the Gell-Mann-Oakes-Renner relation, the low-energy theorems for pion scattering [22] and allowing computations of static pion-nucleon observables [23]. For this baryon sector application we employ the linear confining potential, and neglect all magnetic interactions. This makes the Δ-nucleon mass splitting too small, but does not affect the Δ^* spectrum much.

The model statement equivalent to the generic eq. (9) reads

[2] If, for instance as with a linear potential, the quark mass decreases with momentum with a quartic power-law potential, then the splitting $|M^+ - M^-|$ scales like $\frac{1}{j^3}$. If, on the contrary, the quark mass is constant, and still the potential remains chiral invariant, then the decrease follows a slower power law j^{-1}.

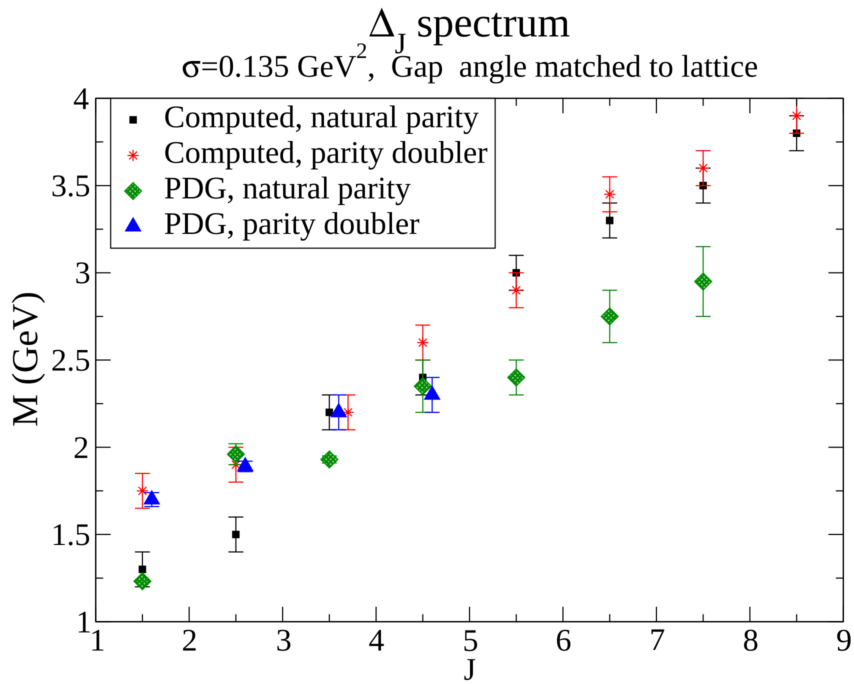

Fig. 3. Parity doubling in the spin-excited Δ spectrum. A three-quark variational Montecarlo computation of the resonance masses in a chiral Hamiltonian inspired in Coulomb-gauge QCD, and that naturally extends the Cornell potential model to a field theory, shows that the ground-states with parity + and − for each $j = 3/2 \dots 13/2$ quickly degenerate. The experimental situation is still unclear, the degeneracy can be claimed for the 9/2 states alone, and the chiral partners higher in the spectrum are not experimentally known.

$$M_+ - M_- = 3 \int \frac{d^3k_1}{(2\pi)^3} \frac{d^3k_2}{(2\pi)^3} \left(\frac{2}{3}\right) \int \frac{d^3q}{(2\pi)^3} \hat{V}(q) \frac{1}{2} \left(\frac{m(|\mathbf{k}_1|)}{|\mathbf{k}_1|} + \frac{m(|\mathbf{k}_1+\mathbf{q}|)}{|\mathbf{k}_1+\mathbf{q}|} \right) \quad (17)$$
$$\times \left[F^{*\lambda_1\lambda_2\lambda_3}(\mathbf{k}_1, \mathbf{k}_2) \left(\mathbb{I} - \sigma \hat{\mathbf{k}}_1 \sigma \widehat{\mathbf{k}_1 + \mathbf{q}} \right)_{\lambda_1\mu_1} F^{\mu_1\lambda_2\lambda_3}(\mathbf{k}_1+\mathbf{q}, \mathbf{k}_2 - \mathbf{q}) \right]$$

while the weaker degeneracy within the quartet is due to a spin-independent potential, two j-powers lower than the leading centrifugal barrier. [3]

We truncate the Fock space variationally, as customary, to the $|qqq\rangle$ minimum wavefunction. Since radial excitations of this system compete with multiquark excitations, we concentrate instead on maximum angular-momentum excitations. One hopes to reduce the molecular component[10] by studying the

[3] The same chirally-invariant quark model was recently applied by Glozman and Wagenbrunn [4] to study insensitivity to spontaneous χSB in the meson spectrum. We can retrospectively understand their numerical results, with similar chiral quartets and j-scaling of the splittings. Indeed their splitting inside the parity doublets tends to zero like $\frac{1}{j^3}$ approximately. A second and weaker degeneracy seems to occur within two of their doublets and this interdoublet splitting tends to zero like $\frac{1}{j^{3/2}}$, as expected in our corollary for spin-independent potentials.

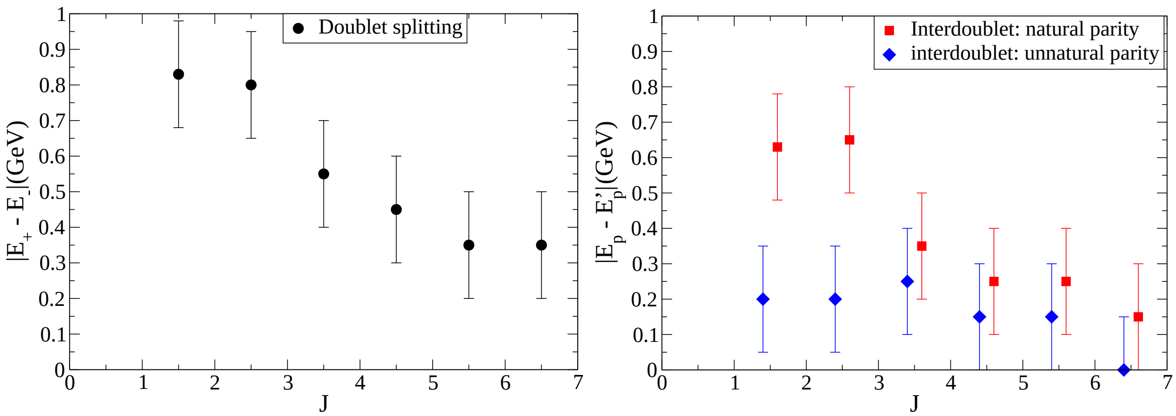

Fig. 4. All model quartet splittings vanish high in the spectrum. Left: splitting inside the first parity doublet. Right: interdoublet splittings between natural and unnatural parity states. The last, higher in the spectrum, is smaller from the start.

ground state in each J-channel, so that the $|qqq\rangle$ correlation remains important high in the spectrum.

We proceed variationally and employ several types of wavefunctions, rational and Gaussian, but the lowest energy (binding the model's j-ground state from above by the Rayleigh-Ritz principle) is obtained by employing the chiral-limit pion-wavefunction rescaled with two variational parameters in terms of the two Jacobi coordinates, $\sin\phi(k_\rho/\alpha_\rho)\sin\phi(k_\lambda/\alpha_\lambda)Y_l^l(\hat{k}_\rho)\chi_{s_1s_2s_3}$, upon which we act the antisymmetrizer of the quark wavefunction, mixing the ρ and λ variables.

As can be seen from figure 4, the model quartet splittings drop with the orbital angular momentum j as predicted with the scalings we derived analytically within our variational Montecarlo error.

6 Phenomenological consequences

To conclude this work, let us look ahead to what the highly excited spin spectrum may reveal. If precise data becomes available at ELSA or Jefferson Lab for the Δ_J with $J = 7/2, 9/2, 11/2, 13/2 \cdots$ parity quartets, one should be able to distinguish between the $1/\sqrt{j}$ fall of $|M^+ - M^-|$ for non-chiral models with a constant difference $(M^+ - M^-)^2$ between the Regge Trajectories [12], and the faster drops for chiral theories such as QCD. This should further motivate analysis of empirical data[24] such as EBAC (Excited Baryon Analysis Center) at Jefferson Lab. In the same way, this should further motivate the different Lattice QCD collaborations who are developing the necessary techniques to compute the baryon spectrum on the lattice.

Since the two doublets are closely degenerate, both positive and negative parity ground states will have a nearby resonance with identical quantum numbers. Given the width of those states, it is likely they will only be distinguished by very careful exclusive decay analysis.

If the excited Δ spectrum could be measured high enough to go beyond the infrared quark mass enhancement, and a lattice calculation of $\langle H_\chi^{QCD\,\prime} \rangle$ became available, an almost direct measurement of the current quark mass follows. In the ultraviolet, the quark mass runs with a slower logarithm, $\bar{m}(Q^2) = \frac{\hat{m}}{\left(\frac{1}{2}\log Q^2/\Lambda^2\right)^{d_m}}$ [25,26] instead of a power-law. The splittings, small by then, do decrease slower.

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