Constructing Observables for Strangeness Production.

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(January 27, 2004)

Contents

1 Introduction

We aim at describing the production of particles that have a non-vanishing "strange" valence contribution. One of the most accessible reactions to reach this goal, is the the "kaon-Y" production on the nucleon, where Y can be either a Λ or a Σ:

$$\left\{ \begin{array}{lcl} \gamma^{(*)} + p & \rightarrow & K^+ + \Lambda^0 \\ \gamma^{(*)} + p & \rightarrow & K^+ + \Sigma^0 \\ \gamma^{(*)} + p & \rightarrow & K^0 + \Sigma^+ \\ \gamma^{(*)} + n & \rightarrow & K^0 + \Lambda^0 \\ \gamma^{(*)} + n & \rightarrow & K^0 + \Sigma^0 \\ \gamma^{(*)} + n & \rightarrow & K^+ + \Sigma^- \end{array} \right.$$

and where $\gamma^{(*)}$ is a real (γ) or virtual (γ^*) photon.

Starting from an initial state where strange valence quarks are absent, the production process is forced to make a connection to the quark sea. After all, to create strangeness, the photon coupling has to rearrange a virtual $s\bar{s}$-quark pair into the final hadronic states. By studying this processes, one hopes to learn more about virtual quark pairs that build up the quark sea. This rearrangement picture is the guideline for the description of the reaction mechanism in models based on the quark picture.

Alternatively, $p(\gamma, K)Y$ reactions can also be described from a hadronic point of view. Then, the degrees of freedom are baryons, mesons and their excited states. In lowest order, the reaction mechanism proceeds through the exchange of intermediate states, or so called resonances. This is schematically depicted in Fig.(4). In what follows, we want to nail down the important resonances in the reaction mechanism of strangeness production and determine the coupling constants corresponding with these exchanged particles. Since long, it is known that the nucleon has excited states but a clear classification of its "spectrum" is still not available. Most of the known nucleon resonances are observed in πN scattering and for some of them, decay into the strange channels has been identified (see Table (11)). Quark model calculations predict a wealth of hadronic resonances, much more than characterized by the πN scattering experiments. Some of this "missing" resonances can possibly play a role in the strangeness production channels.

The ultimate goal of this work is to solve these problems. There for, we have developed a computer code `strangecalc` that calculates observables for $p(\gamma, K)Y$ processes, starting from the Feynman graphs contained in Fig.(4). `strangecalc` is able to calculate differential and total cross sections as well as single and double polarization asymmetries for polarized photons, nucleon targets and recoil hyperons. Photon as well as electron induced processes are computable.

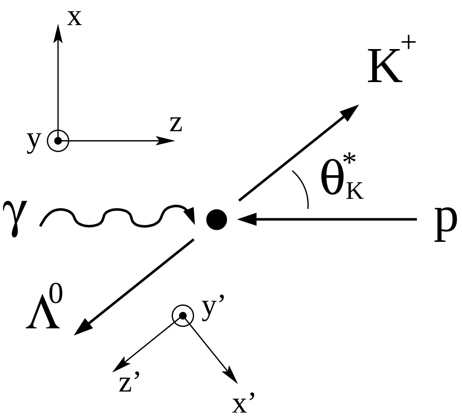

Figure 1: The kinematic conventions for the photoproduction process.

2 The Kinematics

2.1 Photoproduction

In the case of photoproduction we deal with the following reaction where the corresponding four momenta of the particles are given between brackets:

$$p\,(p) + \gamma\,(k) \to K\,(p_K) + Y\,(p_Y)\,. \tag{1}$$

To calculate the differential cross section, we start from the well known expression [8]:

$$d\sigma = \int \frac{1}{v_{rel} 2\omega 2E_p} \frac{d^3\vec{p}_K}{(2\pi)^3} \frac{1}{2E_K} \frac{d^3\vec{p}_Y}{(2\pi)^3} \frac{1}{2E_Y} (2\pi)^4 \delta^{(4)}(p + k - p_K - p_Y) \frac{1}{4} \sum_{\lambda_1 \lambda_2 \lambda} \left| \mathcal{M}_{\lambda}^{\lambda_1 \lambda_2} \right|^2 \,. \tag{2}$$

Here λ is the photon polarization, λ_1 the proton polarization and λ_2 the hyperon polarization. The kinematic conventions are depicted in Fig. 2.1.

2.1.1 The Lab System

In the lab system, the components of the four momenta are:

$$\begin{aligned} p^\mu &= (M_p, 0)\ , & p_K^\mu &= (E_K, \vec{p}_K)\ , \\ k^\mu &= (\omega, \vec{k})\ , & p_Y^\mu &= (E_Y, \vec{p}_Y)\ , \end{aligned} \tag{3}$$

with $|\vec{k}| = \omega$ since the photons are real and they obey the mass relation $k^2 = 0$.
Evaluating the expression (2) in this frame gives:

$$\left.\frac{d\sigma}{d\Omega_K}\right|_{lab} = \frac{1}{64\pi^2} \frac{|\vec{p}_K|}{\omega} \frac{1}{M_p^2} \frac{1}{1 + \frac{\omega}{M_p} - \frac{E_K}{M_p} \frac{\omega}{|\vec{p}_K|} \cos\theta_K} \frac{1}{4} \sum_{\lambda_1 \lambda_2 \lambda} \left| \mathcal{M}_{\lambda}^{\lambda_1 \lambda_2} \right|^2 \ , \tag{4}$$

where E_K and E_Y are a function of ω and θ_K through the relations:

$$E_K = \sqrt{|\vec{p}_K|^2 + M_K^2}\ , \qquad E_Y = \sqrt{M_Y^2 + \omega^2 + |\vec{p}_K|^2 - 2\omega\,|\vec{p}_K|\cos\theta_K}\ , \tag{5}$$

and $|\vec{p}_K|$ is implicitly determined by the energy conservation relation:

$$\omega^* + M_p = \sqrt{M_K^2 + |\vec{p}_K|^2} + \sqrt{M_Y^2 + \omega^2 + |\vec{p}_K|^2 - 2\omega\,|\vec{p}_K|\cos\theta_K}\ . \tag{6}$$

By inspecting Eq.(6), it is clear that, for certain values of ω and θ_K, $|\vec{p}_K|$ does either not exist or has two solutions. This means that the cross section is not uniquely determined by ω and θ_K. We can avoid this problem by working in the "center of mass" frame.

2.1.2 The Center of Mass System

In the c.m. system, the components of the four momenta are:

$$\begin{aligned} p^\mu &= (E_p^*, -\vec{k}^*)\ , & p_K^\mu &= (E_K^*, \vec{p}_K^{\,*})\ , \\ k^\mu &= (\omega^*, \vec{k}^*)\ , & p_Y^\mu &= (E_Y^*, -\vec{p}_K^{\,*})\ , \end{aligned} \tag{7}$$

with $|\vec{k}^*| = \omega^*$ since the mass relation for real photons.
In the center of mass frame, Eq.(2) becomes:

$$\left.\frac{d\sigma}{d\Omega_K^*}\right|_{cm} = \frac{1}{64\pi^2}\frac{|\vec{p}_K^{\,*}|}{\omega^*}\frac{1}{\left(E_p^* + \omega^*\right)^2}\frac{1}{4}\sum_{\lambda_1\lambda_2\lambda}\left|\mathcal{M}_\lambda^{\lambda_1\lambda_2}\right|^2 , \tag{8}$$

where E_p^*,E_K^* and E_Y^* are given by the mass relations:

$$E_p^* = \sqrt{M_p^2 + |\omega^*|^2}\ , \qquad E_K^* = \sqrt{M_K^2 + |\vec{p}_K^{\,*}|^2}\ , \qquad E_Y^* = \sqrt{M_Y^2 + |\vec{p}_K^{\,*}|^2}\ . \tag{9}$$

$|\vec{p}_K^{\,*}|$ is now uniquely determined by the energy conservation relation:

$$\omega^* + \sqrt{M_p^2 + \omega^{*2}} = \sqrt{M_K^2 + |\vec{p}_K^{\,*}|^2} + \sqrt{M_Y^2 + |\vec{p}_K^{\,*}|^2}\ . \tag{10}$$

2.1.3 Lorentz Transformation

The two frames are linked to each other by a Lorentz transformation which reads:

$$\omega = \frac{\omega^*}{M_p}\left[\omega^* + \sqrt{M_p^2 + \omega^{*2}}\right] = \omega^*\frac{W}{M_p} \quad \text{or} \quad \omega^* = \omega\frac{M_p}{\sqrt{(\omega + M_p)^2 - \omega^2}} \tag{11}$$

$$\mathrm{tg}\,(\theta_K) = \frac{1}{\gamma}\frac{\sin(\theta_K^*)}{\cos(\theta_K^*) + \tau}\ . \tag{12}$$

Herein is:

$$\gamma = \frac{1}{\sqrt{1 - V_{lab}^2}}\ , \qquad V_{lab} = \frac{\omega}{M_p + \omega}\ , \qquad \tau = \frac{V_{lab}}{\nu}\ , \tag{13}$$

with $W = \sqrt{s}$ the total invariant mass of the system and ν the velocity of the particle in the c.m. frame.

2.1.4 The Unpolarized Amplitude

The total amplitude occurs in the cross section as $\left|\mathcal{M}_\lambda^{\lambda_1\lambda_2}\right|^2$. In the case that the polarization of the particles remains undetected, we can write:

$$\frac{1}{4}\sum_{\lambda_1\lambda_2\lambda}\left|\mathcal{M}_\lambda^{\lambda_1\lambda_2}\right|^2 = \frac{1}{4}\sum_{\lambda_1\lambda_2\lambda}\left|\varepsilon_\lambda^\mu J_\mu^{\lambda_1\lambda_2}\right|^2 , \tag{14}$$

$$= \frac{1}{4}\sum_{\lambda_1\lambda_2\lambda}\varepsilon_\lambda^\mu\varepsilon_\lambda^{\nu*}J_\mu^{\lambda_1\lambda_2}J_\nu^{\lambda_1\lambda_2\dagger} . \tag{15}$$

The summation over the photon polarization λ becomes very simple if the amplitude is gauge invariant. If there holds $k^\mu J_\mu = 0$, we can make the replacement: $\sum_{\lambda=\pm1}\varepsilon_\lambda^\mu\varepsilon_\lambda^{\nu*} \to -g^{\mu\nu}$. The current J_μ is conserved as far as there are no form factors introduced. The three Born terms together preserve gauge invariance and the remaining terms do this on their own. So, we can write:

$$\frac{1}{4}\sum_{\lambda_1\lambda_2\lambda}\left|\mathcal{M}_\lambda^{\lambda_1\lambda_2}\right|^2 = -\frac{g^{\mu\nu}}{4}\sum_{\lambda_1\lambda_2}J_\mu^{\lambda_1\lambda_2}J_\nu^{\lambda_1\lambda_2\dagger} , \tag{16}$$

or

$$\frac{1}{4}\sum_{\lambda_1\lambda_2\lambda}\left|\mathcal{M}_\lambda^{\lambda_1\lambda_2}\right|^2 = -\frac{g^{\mu\nu}}{4}\sum_{\lambda_1\lambda_2}\overline{U}_Y^{\lambda_2}(p_Y)\,T_\mu U_p^{\lambda_1}(p)\,\overline{U}_p^{\lambda_1}(p)\,\overline{T}_\nu U_Y^{\lambda_2}(p_Y) . \tag{17}$$

Herein is T the "amputated" current, i.e. the spinors of the proton and hyperon field are removed. $\overline{T}_\mu$ is defined as $\gamma^0 T_\mu^\dagger \gamma^0$. Using some properties of the Dirac algebra, the summation can be calculated as:

$$\frac{1}{4}\sum_{\lambda_1\lambda_2\lambda}\left|\mathcal{M}_\lambda^{\lambda_1\lambda_2}\right|^2 = -\frac{1}{4}\mathrm{Tr}\left\{(\not{p}_Y + M_Y)\,T_\mu\,(\not{p} + M_p)\,\overline{T}^\mu\right\} . \tag{18}$$

The result is a function of four vector products like $(p\cdot k)$, $(p_K\cdot k)$, $(p\cdot p_Y)$ ... which all are function of ω and θ_K. The explicit expressions, evaluated in the c.m. frame, read:

$$(p\cdot k) = \omega^*\left(\sqrt{M_p^2+\omega^{*2}}+\omega^*\right) , \tag{19}$$

$$(p_K\cdot k) = \omega^*\left(\sqrt{M_K^2+|\vec{p}_K^{\,*}|^2}-|\vec{p}_K^{\,*}|\cos\theta_K^*\right) , \tag{20}$$

$$(p_Y\cdot k) = \omega^*\left(\sqrt{M_Y^2+|\vec{p}_K^{\,*}|^2}+|\vec{p}_K^{\,*}|\cos\theta_K^*\right) , \tag{21}$$

$$(p\cdot p_K) = \sqrt{M_p^2+\omega^{*2}}\sqrt{M_K^2+|\vec{p}_K^{\,*}|^2}+\omega^*|\vec{p}_K^{\,*}|\cos\theta_K^* , \tag{22}$$

$$(p\cdot p_Y) = \sqrt{M_p^2+\omega^{*2}}\sqrt{M_Y^2+|\vec{p}_K^{\,*}|^2}-\omega^*|\vec{p}_K^{\,*}|\cos\theta_K^* , \tag{23}$$

$$(p_Y\cdot p_K) = \sqrt{M_Y^2+|\vec{p}_K^{\,*}|^2}\sqrt{M_K^2+|\vec{p}_K^{\,*}|^2}+|\vec{p}_K^{\,*}|^2 . \tag{24}$$

where $|\vec{p}_K^{\,*}|$ is implicit given by Eq.(10). In this way, we have a closed form for the differential cross section of the photoproduction process as a function of ω^* and θ_K^*.

2.1.5 Photon Polarization

To describe polarized photoproduction, one has to realize that a real photon has only two independent polarization vectors.
For linearly polarized photons along the $\vec{x}$ and $\vec{y}$ axis, this polarization vectors read:

$$\varepsilon^{\lambda=+1} = (0,1,0,0) \ , \qquad\qquad \varepsilon^{\lambda=-1} = (0,0,1,0) \ . \tag{25}$$

For circularly polarized photons, this becomes:

$$\varepsilon^{\lambda=+1} = -\frac{1}{\sqrt{2}}(0,1,i,0) \ , \qquad\qquad \varepsilon^{\lambda=-1} = \frac{1}{\sqrt{2}}(0,1,-i,0) \ . \tag{26}$$

Here, we have chosen the $\vec{z}$-axis along the photon momentum, the $\vec{y}$-axis perpendicular to the reaction plane and the $\vec{x}$-axis in the reaction plane perpendicular to $\vec{y}$ and $\vec{z}$.
Since one of the polarizations is known, we have to make the replacement:

$$\frac{1}{4}\sum_{\lambda_1\lambda_2\lambda}\left|\mathcal{M}_{\lambda}^{\lambda_1\lambda_2}\right|^2 \rightarrow \frac{1}{2}\sum_{\lambda_1\lambda_2}\left|\mathcal{M}_{\lambda=\pm1}^{\lambda_1\lambda_2}\right|^2 = \frac{1}{2}\sum_{\lambda_1\lambda_2}\left|\varepsilon^{\mu}_{\lambda=\pm1}J_{\mu}^{\lambda_1\lambda_2}\right|^2 \ . \tag{27}$$

In analogy with Eq.(18) we can cast this spin sum in a trace calculation:

$$\frac{1}{2}\sum_{\lambda_1\lambda_2}\left|\mathcal{M}_{\lambda=\pm1}^{\lambda_1\lambda_2}\right|^2 = \frac{1}{2}\mathrm{Tr}\left\{(\not{p}_Y + M_Y)\,T^{\mu}\varepsilon^{\lambda}_{\mu}\,(\not{p}+M_p)\,\overline{T}^{\nu}\varepsilon^{\lambda *}_{\nu}\right\}\Bigg|_{\lambda=\pm1} \ . \tag{28}$$

The result of this calculation is a function of the four vector products in Eq.(20-24) and of four vector products of the external four momenta and the photon polarization vectors.

	linearly pol. photons	circularly pol. photons
$(\varepsilon^{\lambda}\cdot k)$	0	0
$(\varepsilon^{\lambda}\cdot p)$	0	0
$(\varepsilon^{\lambda}\cdot p_K)$	$-\lvert\vec{p}_K^{\,*}\rvert\sin\theta_K^*\ \delta_{\lambda,+1}$	$\frac{\lambda}{\sqrt{2}}\lvert\vec{p}_K^{\,*}\rvert\sin\theta_K^*$
$(\varepsilon^{\lambda}\cdot p_Y)$	$\lvert\vec{p}_K^{\,*}\rvert\sin\theta_K^*\ \delta_{\lambda,+1}$	$-\frac{\lambda}{\sqrt{2}}\lvert\vec{p}_K^{\,*}\rvert\sin\theta_K^*$
$(\varepsilon^{\lambda}\cdot\varepsilon^{\lambda' *})$	$-\delta_{\lambda,\lambda'}$	$-\delta_{\lambda,\lambda'}$

(29)

Since the parity conservation of the Lagrangian, the reaction plane is a symmetry plane but the parity operator converts a left handed polarized photon is in a right handed photon and vice versa. From this it is clear that there can be no asymmetry for circularly polarized photons.
The (linearly) polarized photon beam asymmetry Σ is then defined as:

$$\Sigma = \frac{d\sigma/d\Omega^{(\perp)} - d\sigma/d\Omega^{(\parallel)}}{d\sigma/d\Omega^{(\perp)} + d\sigma/d\Omega^{(\parallel)}} \ , \tag{30}$$

where $d\sigma/d\Omega^{(\parallel,\perp)}$ corresponds to $\mathcal{M}^{\lambda_1\lambda_2}_{\lambda=\pm1}$.

2.1.6 Target Polarization

To describe target polarization, this is polarization of the proton, we can use the following mathematical tricks to fall back on our standard trace calculation program.
In expression (8) for the unpolarized differential cross section we have to make the replacement:

$$\frac{1}{4}\sum_{\lambda_1\lambda_2\lambda}\left|\mathcal{M}^{\lambda_1\lambda_2}_{\lambda}\right|^2 \to \frac{1}{2}\sum_{\lambda_2\lambda}\left|\mathcal{M}^{\lambda_1=\pm1,\lambda_2}_{\lambda}\right|^2 \ . \tag{31}$$

Using the considerations before Eq.(16) we can write:

$$\frac{1}{2}\sum_{\lambda_2\lambda}\left|\mathcal{M}^{\lambda_1=\pm1,\lambda_2}_{\lambda}\right|^2 = -\frac{g^{\mu\nu}}{2}\sum_{\lambda_2}\overline{U}^{\lambda_2}_{Y}(p_Y)\,T_\mu U^{\lambda_1=\pm1}_{p}(p)\,\overline{U}^{\lambda_1=\pm1}_{p}(p)\,\overline{T}_\nu U^{\lambda_2}_{Y}(p_Y)\ , \tag{32}$$

$$= -\frac{g^{\mu\nu}}{2}\sum_{\lambda_1\lambda_2}\overline{U}^{\lambda_2}_{Y}(p_Y)\,T_\mu \Pi^{\pm}(n_p)\,U^{\lambda_1}_{p}(p)\,\overline{U}^{\lambda_1}_{p}(p)\,\left(\Pi^{\pm}(n_p)\right)^{\dagger}\overline{T}_\nu U^{\lambda_2}_{Y}(p_Y)\ . \tag{33}$$

Here, we have introduced the spin projection operator:

$$\Pi^{\pm}(n_p) = \frac{1}{2}\left(1 \pm \gamma_5\ \not{n}_p\right)\ , \tag{34}$$

with the four vector n^{μ}_p in the rest frame of the proton defined as $n^{\mu}_p = (0, \vec{n}_p)$ and $\vec{n}_p$ the spin quantization axis. From these properties, there holds in any other frame $n^2_p = -1$ and $(n_p \cdot p) = 0$. Using some properties of this projection operator, we can write:

$$\frac{1}{2}\sum_{\lambda_2\lambda}\left|\mathcal{M}^{\lambda_1=\pm1,\lambda_2}_{\lambda}\right|^2 = -\frac{1}{4}\mathrm{Tr}\left\{(\not{p}_Y + M_Y)\,T_\mu\,(1+\lambda_1\gamma_5\ \not{n}_p)\,(\not{p}+M_p)\,\overline{T}^{\mu}\right\}\Big|_{\lambda_1=\pm1}\ . \tag{35}$$

The result of this trace calculation is a function of the four vector products in Eq.(20-24) and of four vector products of the vector n_p in combination with one of the external four vectors. The spin quantization axis $\vec{n}_p$ is chosen perpendicular to the reaction plane and after a Lorentz boost from the rest frame of the proton (= lab frame) to the c.m. frame this n_p remains unchanged. So, every four vector product that includes n_p, will vanish.
With the inclusion of the spin projection operator, it becomes possible that a single γ_5 remains in the trace. In this cases, we have to fall back on the formula:

$$\mathrm{Tr}\left\{\gamma^5\right\} = \mathrm{Tr}\left\{\gamma^\alpha\gamma^5\right\} = \mathrm{Tr}\left\{\gamma^\alpha\gamma^\beta\gamma^5\right\} = \mathrm{Tr}\left\{\gamma^\alpha\gamma^\beta\gamma^\mu\gamma^5\right\} = 0\ , \tag{36}$$

and:

$$\mathrm{Tr}\left\{\gamma^\alpha\gamma^\beta\gamma^\mu\gamma^\nu\gamma^5\right\} = -4\,i\,\epsilon^{\alpha\beta\mu\nu}\ , \tag{37}$$

or alternatively:

$$\begin{aligned} \mathrm{Tr}\{\not{a}\ \not{b}\ \not{c}\ \not{d}\gamma_5\} &= -4\,i\ \det(a\,b\,c\,d)\ , \\ &= -4\,i \begin{vmatrix} a^0 & b^0 & c^0 & d^0 \\ -a^1 & -b^1 & -c^1 & -d^1 \\ -a^2 & -b^2 & -c^2 & -d^2 \\ -a^3 & -b^3 & -c^3 & -d^3 \end{vmatrix}\ . \end{aligned} \tag{38}$$

Remark that, since ϵ^{0123} is defined positive in a contra variant manner, the four vectors in the determinant have to be covariant!
In our calculations, we arrive with traces as:

$$\begin{aligned} \mathrm{Tr}\{\not{n}_p\ \not{k}\ \not{p}\ \not{p}_K\gamma_5\} &= -4\,i\ \det(n_p\ k\ p\ p_K)\ , \\ &= -4\,i \begin{vmatrix} 0 & \omega^* & E_p^* & E_K^* \\ 0 & 0 & 0 & -|\vec{p}_K^*|\sin\theta_K^* \\ -1 & 0 & 0 & 0 \\ 0 & -\omega^* & \omega^* & -|\vec{p}_K^*|\cos\theta_K^* \end{vmatrix}\ , \\ &= 4\,i\,\omega^*\,|\vec{p}_K^*|\ \sin\theta_K^* \left(\sqrt{M_p^2+\omega^{*2}}+\omega^*\right)\ . \end{aligned} \tag{39}$$

The polarized proton target asymmetry T is defined as:

$$T = \frac{d\sigma/d\Omega^{(+)} - d\sigma/d\Omega^{(-)}}{d\sigma/d\Omega^{(+)} + d\sigma/d\Omega^{(-)}}\ , \tag{40}$$

where $+\,(-)$ refers to proton polarization parallel (anti parallel) to the $\vec{y} = (\vec{k}\times\vec{p}_K)/|\vec{k}\times\vec{p}_K$ axis. Remark that the only no-vanishing determinant is achieved for $\vec{n}_p$ pointing along the $\vec{y}$-axis. All the other polarization asymmetries are zero.

In order to calculate the asymmetry, it is sufficient only to calculate the asymmetric part of the polarized cross section. In a simplified notation, one can write

$$\langle\sigma^{(\pm)}\rangle = \langle\sigma\rangle \pm \langle\gamma_5\rangle\ , \tag{41}$$

where $\langle\sigma\rangle$ is the unpolarized cross section and $\langle\gamma_5\rangle$ as the asymmetric part. In this notation, the averaging (a factor 1/2) over the initial photon spin and the factor 1/2 of the spin projection operator are included. Since in the unpolarized cross section is averaged over the initial target spins, we can write:

$$\langle\sigma\rangle = \frac{\langle\sigma^{(+)}\rangle + \langle\sigma^{(-)}\rangle}{2}\ . \tag{42}$$

Now it is easy to prove that Eq. (40) reduces to:

$$T = \frac{\langle\gamma_5\rangle}{\langle\sigma\rangle}\ . \tag{43}$$

2.1.7 Recoil Polarization

For recoil polarization, or in this case, hyperon polarization, we can use the same techniques as outlined in subsection (2.1.6). Along the same lines we arrive at the formula:

$$\frac{1}{4}\sum_{\lambda_1\lambda}\left|\mathcal{M}_{\lambda}^{\lambda_1,\lambda_2=\pm1}\right|^2 = -\frac{1}{8}\mathrm{Tr}\left\{(1+\lambda_2\gamma_5\ \not{n}_Y)\,(\not{p}_Y+M_Y)\,T_\mu\,(\not{p}+M_p)\,\overline{T}^\mu\right\}\Big|_{\lambda_2=\pm1}\ , \tag{44}$$

where the vector n_Y obey the same properties as n_p is section (2.1.6).
The hyperon polarization asymmetry P is defined by:

$$P = \frac{d\sigma/d\Omega^{(+)} - d\sigma/d\Omega^{(-)}}{d\sigma/d\Omega^{(+)} + d\sigma/d\Omega^{(-)}}\ , \tag{45}$$

where $+\,(-)$ refers to hyperon polarization parallel (anti parallel) to the $\vec{y}$ axis. Here again, the polarized cross section is given in a short hand notation by:

$$\langle\sigma^{(\pm)}\rangle = \frac{1}{2}\langle\sigma\rangle \pm \langle\gamma_5\rangle\ . \tag{46}$$

For the recoil polarization case, the unpolarized cross section is the sum of the two polarized parts. So,

$$\langle\sigma\rangle = \langle\sigma^{(+)}\rangle + \langle\sigma^{(-)}\rangle\ , \tag{47}$$

and the polarization asymmetry can be rewritten as:

$$P = \frac{2\langle\gamma_5\rangle}{\langle\sigma\rangle}\ . \tag{48}$$

2.1.8 Double Polarization Asymmetries

The definition of the double polarization asymmetry observables is given by:

$$X = \frac{d\sigma/d\Omega^{(++)} + d\sigma/d\Omega^{(--)} - d\sigma/d\Omega^{(+-)} - d\sigma/d\Omega^{(-+)}}{d\sigma/d\Omega^{(++)} + d\sigma/d\Omega^{(--)} + d\sigma/d\Omega^{(+-)} + d\sigma/d\Omega^{(-+)}} \tag{49}$$

with X one of the 12 observables listed in Table(1). The $+\,(-)$ refers to a polarization parallel (anti parallel) to the respective quantization axis or helicity state (in the case of circularly polarized photons).
The particular choice of the quantization axes (Table(1)) allows to rewrite Eq.(49):

$$\begin{aligned} X &= \frac{d\sigma/d\Omega^{(++)} - d\sigma/d\Omega^{(+-)}}{d\sigma/d\Omega^{(++)} + d\sigma/d\Omega^{(+-)}}\ , \\ &= \frac{d\sigma/d\Omega^{(--)} - d\sigma/d\Omega^{(-+)}}{d\sigma/d\Omega^{(--)} + d\sigma/d\Omega^{(-+)}}\ , \end{aligned} \tag{50}$$

Observable	γ	p	Λ
beam-target			
E	c	$-z$	
F	c	$-x$	
G	t	$-z$	
H	t	$-x$	
beam-recoil			
C_x	c		x'
C_z	c		z'
O_x	t		x'
O_z	t		z'
target-recoil			
T_x		$-x$	x'
T_z		$-x$	z'
L_x		$-z$	x'
L_z		$-z$	z'

Table 1: Definition of the double polarization observables. Quantization axes are defined as follows: $\vec{z} \sim \vec{k}$, $\vec{y} \sim (\vec{k} \times \vec{p}_K)$, $\vec{x} = \vec{y} \times \vec{z}$, $\vec{z}' \sim \vec{p}_\Lambda$, $\vec{y} = \vec{y}'$, $\vec{x}' = \vec{y}' \times \vec{z}'$. t $\rightarrow$ linearly polarized photon ($\pm\pi/4$ with respect to scattering plane); c $\rightarrow$ circularly polarized photon.

since $d\sigma/d\Omega^{(++)} = d\sigma/d\Omega^{(--)}$ and $d\sigma/d\Omega^{(+-)} = d\sigma/d\Omega^{(-+)}$.

Calculating this double polarization observables, for every combination of polarized particles, one ends up with a specific traces.

Before specifying each particular combination, we report six general relations among these double polarization observables [2], [3]:

$$E^2 + F^2 + G^2 + H^2 = 1 + P^2 - \Sigma^2 - T^2 \quad (51)$$

$$FG - EH = P - \Sigma T \quad (52)$$

$$C_x^2 + C_z^2 + O_x^2 + O_z^2 = 1 + T^2 - P^2 - \Sigma^2 \quad (53)$$

$$C_z O_x - C_x O_z = T - P\Sigma \quad (54)$$

$$T_x^2 + T_z^2 + L_x^2 + L_z^2 = 1 + \Sigma^2 - P^2 - T^2 \quad (55)$$

$$T_x L_z - T_z L_x = \Sigma - PT \quad (56)$$

Beam-Target:

$$\frac{1}{4}\sum_{\lambda_1\lambda_2\lambda}\left|\mathcal{M}_{\lambda}^{\lambda_1\lambda_2}\right|^2 \to \sum_{\lambda_2}\left|\mathcal{M}_{\lambda=\pm1}^{\lambda_1=\pm1,\lambda_2}\right|^2 = \frac{1}{2}\mathrm{Tr}\left\{(\not{p}_Y + M_Y)\,T^{\mu}\varepsilon_{\mu}^{\lambda}\,(1+\lambda_1\gamma_5\,\not{n}_p)\,(\not{p}+M_p)\,\overline{T}^{\nu}\,\varepsilon_{\nu}^{\lambda *}\right\}\Big|_{\lambda,\lambda_1=\pm1}, \quad (57)$$

where the four vectors n_p and ε^{λ} have the explicit expression:

X	n_p	$\varepsilon^{\lambda=\pm1}$
E	$\frac{1}{M_p}\left(\omega^*, 0, 0, -E_p^*\right)$	$\frac{\mp}{\sqrt{2}}(0, 1, \pm i, 0)$
F	$(0, -1, 0, 0)$	$\frac{\mp}{\sqrt{2}}(0, 1, \pm i, 0)$
G	$\frac{1}{M_p}\left(\omega^*, 0, 0, -E_p^*\right)$	$\frac{1}{\sqrt{2}}(0, 1, \pm 1, 0)$
H	$(0, -1, 0, 0)$	$\frac{1}{\sqrt{2}}(0, 1, \pm 1, 0)$

(58)

Remark that n_p is constructed in such a way that it satisfy the relations $(n_p \cdot p) = 0$ and $n_p^2 = -1$, which can easily be proved in the rest frame of the target.

Calculating the beam-target asymmetry, it is sufficient to calculate the unpolarized cross section and the asymmetric part with a polarized photon. The double polarized cross section is in the short hand notation defined in a previous section given by

$$\langle\sigma^{(\pm,\pm)}\rangle = \langle\sigma^{(\pm)}\rangle \pm \langle\gamma_5^{(\pm)}\rangle \; . \tag{59}$$

Since both initial particles are polarized, the unpolarized cross section is given by the average of the four different double polarizations:

$$\langle\sigma\rangle = \frac{\langle\sigma^{(+,+)}\rangle + \langle\sigma^{(+,-)}\rangle + \langle\sigma^{(-,+)}\rangle + \langle\sigma^{(-,-)}\rangle}{4} \; . \tag{60}$$

Eq. (50) reduces now into the expression:

$$X = \frac{\langle\gamma_5^{(+)}\rangle}{\langle\sigma\rangle} \; , \tag{61}$$

where $\langle\gamma_5^{(+)}\rangle$ is the asymmetric part of the cross section, calculated with a photon with positive helicity.

Beam-Recoil:

$$\begin{aligned} \frac{1}{4}\sum_{\lambda_1\lambda_2\lambda}\left|\mathcal{M}_{\lambda}^{\lambda_1\lambda_2}\right|^2 &\to \frac{1}{2}\sum_{\lambda_1}\left|\mathcal{M}_{\lambda=\pm 1}^{\lambda_1,\lambda_2=\pm 1}\right|^2 \\ &= \frac{1}{4}\mathrm{Tr}\left\{(1+\lambda_2\gamma_5\;\not{n}_Y)\,(\not{p}_Y + M_Y)\,T^{\mu}\varepsilon_{\mu}^{\lambda}\,(\not{p} + M_p)\,\overline{T}^{\nu}\varepsilon_{\nu}^{\lambda *}\right\}\Big|_{\lambda,\lambda_2=\pm 1} \; , \end{aligned} \tag{62}$$

where the four vectors n_Y and ε^{λ} have the explicit expression:

X	n_Y	$\varepsilon^{\lambda=\pm 1}$
C_x	$(0, -\cos\theta_K^*, 0, \sin\theta_K^*)$	$\frac{\mp}{\sqrt{2}}\,(0, 1, \pm i, 0)$
C_z	$\frac{1}{M_Y}\,(\lvert\vec{p}_K^*\rvert, -E_Y^*\sin\theta_K^*, 0, -E_Y^*\cos\theta_K^*)$	$\frac{\mp}{\sqrt{2}}\,(0, 1, \pm i, 0)$
O_x	$(0, -\cos\theta_K^*, 0, \sin\theta_K^*)$	$\frac{1}{\sqrt{2}}\,(0, 1, \pm 1, 0)$
O_z	$\frac{1}{M_Y}\,(\lvert\vec{p}_K^*\rvert, -E_Y^*\sin\theta_K^*, 0, -E_Y^*\cos\theta_K^*)$	$\frac{1}{\sqrt{2}}\,(0, 1, \pm 1, 0)$

(63)

Here again, the four vector n_Y fulfill the relations $(n_Y \cdot p_Y) = 0$ and $n_Y^2 = -1$.

For the beam-recoil asymmetry, the same remark hold as i n the case of the beam-target asymmetry. It is sufficient to calculate the unpolarized cross section and the asymmetric part with a polarized photon. The double polarized cross section is given by

$$\langle\sigma^{(\pm,\pm)}\rangle = \frac{1}{2}\langle\sigma^{(\pm)}\rangle \pm \langle\gamma_5^{(\pm)}\rangle \; . \tag{64}$$

Since one initial particle (the photon) is polarized and one not (the target), the unpolarized cross section is given by partly averaging **and** partly summing over the four different double polarizations:

$$\langle\sigma\rangle = \frac{\langle\sigma^{(+,+)}\rangle + \langle\sigma^{(+,-)}\rangle + \langle\sigma^{(-,+)}\rangle + \langle\sigma^{(-,-)}\rangle}{2} \,. \tag{65}$$

In this case Eq. (50) reduces into:

$$X = \frac{2\langle\gamma_5^{(+)}\rangle}{\langle\sigma\rangle} \,. \tag{66}$$

Target-Recoil:

$$\begin{aligned} \frac{1}{4}\sum_{\lambda_1\lambda_2\lambda}\left|\mathcal{M}_\lambda^{\lambda_1\lambda_2}\right|^2 &\to \frac{1}{2}\sum_{\lambda}\left|\mathcal{M}_\lambda^{\lambda_1=\pm1,\lambda_2=\pm1}\right|^2 \\ &= -\frac{1}{8}\mathrm{Tr}\left\{(1+\lambda_2\gamma_5\ \not{n}_Y)\,(\not{p}_Y + M_Y)\,T^\mu\,(1+\lambda_1\gamma_5\ \not{n}_p)\,(\not{p}+M_p)\,\overline{T}_\mu\right\}\Big|_{\lambda_1,\lambda_2=\pm1} \,. \end{aligned} \tag{67}$$

The four vectors n_p and n_Y have the explicit expression:

X	n_p	n_Y
T_x	$(0,1,0,0)$	$(0,-\cos\theta_K^*,0,\sin\theta_K^*)$
T_z	$(0,1,0,0)$	$\frac{1}{M_Y}\,(\lvert\vec{p}_K^*\rvert,-E_Y^*\sin\theta_K^*,0,-E_Y^*\cos\theta_K^*)$
L_x	$\frac{1}{M_p}\left(\omega^*,0,0,-E_p^*\right)$	$(0,-\cos\theta_K^*,0,\sin\theta_K^*)$
L_z	$\frac{1}{M_p}\left(\omega^*,0,0,-E_p^*\right)$	$\frac{1}{M_Y}\,(\lvert\vec{p}_K^*\rvert,-E_Y^*\sin\theta_K^*,0,-E_Y^*\cos\theta_K^*)$

(68)

Beside the four vector products already listed above, the trace calculations for double polarization observables results in expressions as collected in Table 2. Here n_x can be n_p or n_Y. Remark that, with the choice of polarization axes as in Table 1, some of the determinants are zero.

2.2 Electroproduction

2.2.1 Unpolarized Reactions

In the case of electroproduction we deal with the following reaction where the corresponding four momenta of the particles are given between brackets:

$$p\,(p) + e\,(k_1) \to e'\,(k_2) + K\,(p_K) + Y\,(p_Y) \,. \tag{69}$$

The kinematic conventions are depicted in Fig. 2.2.1. To calculate the differential cross section in

beam-target & beam-recoil	target-recoil
$(n_x \cdot p)$	$(n_x \cdot p)$
$(n_x \cdot k)$	$(n_x \cdot k)$
$(n_x \cdot p_K)$	$(n_x \cdot p_K)$
$(n_x \cdot p_Y)$	$(n_x \cdot p_Y)$
$(n_x \cdot \varepsilon^\lambda)$	$(n_p \cdot n_Y)$
$\det(n_x\ k\ p\ p_K) \equiv 0$	$\det(n_p\ k\ p\ p_K) \equiv 0$
$\det(n_x\ \varepsilon^\lambda\ k\ p)$	$\det(n_Y\ k\ p\ p_K) \equiv 0$
$\det(n_x\ \varepsilon^\lambda\ k\ p_K)$	$\det(n_p\ n_Y\ k\ p) \equiv 0$
$\det(n_x\ \varepsilon^\lambda\ p\ p_K)$	$\det(n_p\ n_Y\ k\ p_K) \equiv 0$
$\det(n_x\ \varepsilon^{\lambda *}\ k\ p)$	$\det(n_p\ n_Y\ p\ p_K) \equiv 0$
$\det(n_x\ \varepsilon^{\lambda *}\ k\ p_K)$	
$\det(n_x\ \varepsilon^{\lambda *}\ p\ p_K)$	
$\det(n_x\ \varepsilon^\lambda\ \varepsilon^{\lambda *}\ k)$	
$\det(n_x\ \varepsilon^\lambda\ \varepsilon^{\lambda *}\ p)$	
$\det(n_x\ \varepsilon^\lambda\ \varepsilon^{\lambda *}\ p_K)$	
$\det(\varepsilon^\lambda\ k\ p\ p_K)$	
$\det(\varepsilon^{\lambda *}\ k\ p\ p_K)$	
$\det(\varepsilon^\lambda\ \varepsilon^{\lambda *}\ k\ p)$	
$\det(\varepsilon^\lambda\ \varepsilon^{\lambda *}\ k\ p_K)$	
$\det(\varepsilon^\lambda\ \varepsilon^{\lambda *}\ p\ p_K)$	

Table 2: The determinants that appear in the double polarization observables. n_x can be n_p or n_Y.

t'

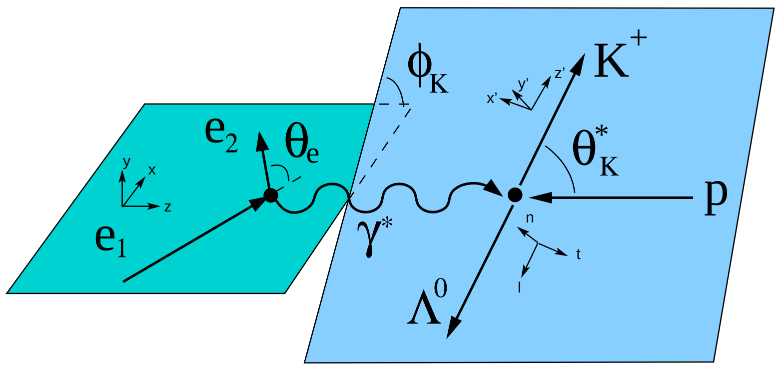

Figure 2: The kinematic conventions for the electroproduction process.

the one photon exchange approximation, we start from the well known expression [8, 17]:

$$d\sigma = \int \frac{1}{v_{rel} 2\epsilon_1 2E_p} \frac{d^3\vec{p}_K}{(2\pi)^3} \frac{1}{2E_K} \frac{d^3\vec{p}_Y}{(2\pi)^3} \frac{1}{2E_Y} \frac{d^3\vec{k}_2}{(2\pi)^3} \frac{1}{2\epsilon_2} (2\pi)^4 \, \delta^{(4)} \left(p + k_1 - p_K - p_Y - k_2\right) \frac{1}{4} \sum_{\lambda' s} |\mathcal{T}|^2 \ . \tag{70}$$

The electron kinematics is solved in the lab system, the hadron kinematics in the center of mass frame of the virtual photon-proton system. With this choice, the components of the four momenta are:

$$\begin{aligned} k_1^\mu &= (\epsilon_1, \vec{k}_1) \ , & k_2^\mu &= (\epsilon_2, \vec{k}_2) \ , & k^\mu &= (\omega, \vec{k}) \\ p^\mu &= (E_p^*, -\vec{k}^*) \ , & p_K^\mu &= (E_K^*, \vec{p}_K^{\,*}) \ , & p_Y^\mu &= (E_Y^*, -\vec{p}_K^{\,*}) \ , \end{aligned} \tag{71}$$

with $k^\mu = k_1^\mu - k_2^\mu$ the four momentum of the virtual photon in the lab frame. Evaluating Eq.(70) leads to:

$$\frac{d\sigma}{d\epsilon_2 d\Omega_2 d\Omega_K^*} = \frac{1}{32\,(2\pi)^5} \frac{1}{M_p} \frac{|\vec{p}_K^{\,*}|}{W} \frac{\epsilon_2}{\epsilon_1} \frac{1}{4} \sum_{\lambda' s} |\mathcal{T}|^2 \ . \tag{72}$$

Herein is the invariant final hadronic mass:

$$\begin{aligned} W &= \sqrt{s} \ , \\ &= E_p^* + \omega^* \ , \\ &= \sqrt{(\omega + M_p)^2 - |\vec{k}|^2} \ , \end{aligned} \tag{73}$$

and are E_p^*, E_K^* and E_Y^* determined as in Eq.(9) with a $|\vec{k}^*|^2$ dependence for E_p^*. $|\vec{p}_K^{\,*}|$ is again implicitly given by the energy conservation relation:

$$w^* + \sqrt{M_p^2 + |\vec{k}^*|^2} = \sqrt{M_K^2 + |\vec{p}_K^{\,*}|^2} + \sqrt{M_Y^2 + |\vec{p}_K^{\,*}|^2} \ , \tag{74}$$

The only coupling between the "lepton lab frame" and the "hadron c.m. frame" is by the virtual photon. The Lorentz transformations of the energy and three momentum of this particle reads:

$$\vec{k}^* = \vec{k}\frac{M_p}{\sqrt{(\omega + M_p)^2 - |\vec{k}|^2}} = \vec{k}\left(\frac{M_p}{W}\right) \ , \tag{75}$$

$$\begin{aligned} \omega^* &= \frac{\omega\,(\omega + M_p) - |\vec{k}|^2}{\sqrt{(\omega + M_p)^2 - |\vec{k}|^2}} \ , \\ &= W - (\omega + M_p)\,\frac{M_p}{W} \ , \\ &= \frac{s - M_p^2 - Q^2}{2W} \ . \end{aligned} \tag{76}$$

The abbreviation of the transition amplitude in Eq.(70) reads as:

$$\frac{1}{4}\sum_{\lambda' s} |\mathcal{T}|^2 = \frac{1}{4}\sum_{\lambda_1\lambda_2\lambda_1'\lambda_2'} \left|\mathcal{T}_{\lambda_1\lambda_2}^{\lambda_1'\lambda_2'}\right|^2 \ , \tag{77}$$

with λ_1 the nucleon and λ_2 the hyperon polarization vectors and λ_1' and λ_2' the initial and final electron polarization.
The transition amplitude can be expressed as a combination of the leptonic current, the photon propagator and the hadronic current.

$$\begin{aligned} \mathcal{T}_{\lambda_1\lambda_2}^{\lambda_1'\lambda_2'} &= e\, l_\mu^{\lambda_1'\lambda_2'}\, \frac{-g^{\mu\nu}}{k^2}\, J_\nu^{\lambda_1\lambda_2} \ , \\ &= \frac{e}{Q^2}\, l_\mu^{\lambda_1'\lambda_2'} \left(g^{\mu\nu} + \frac{k^\mu k^\nu}{Q^2}\right) J_\nu^{\lambda_1\lambda_2} \ , \\ &= \frac{e}{Q^2}\sum_{\lambda=0\pm1} (-1)^\lambda\, L_\lambda^{\lambda_1'\lambda_2' *} \mathcal{M}_\lambda^{\lambda_1\lambda_2} \ , \end{aligned} \tag{78}$$

since current conservation ($k^\mu J_\mu^{\lambda_1\lambda_2} = 0$) and the identity $\sum_{\lambda=0\pm1} (-1)^\lambda\, \varepsilon_\lambda^{\mu *} \varepsilon_\lambda^\nu = \left(g^{\mu\nu} + \frac{k^\mu k^\nu}{Q^2}\right)$.
For the leptonic and hadronic tensors we use the notation:

$$L_\lambda^{\lambda_1'\lambda_2' *} = l_\mu^{\lambda_1'\lambda_2'} \varepsilon_\lambda^{\mu *} \ , \qquad \mathcal{M}_\lambda^{\lambda_1\lambda_2} = J_\mu^{\lambda_1\lambda_2} \varepsilon_\lambda^\mu \ . \tag{79}$$

For the virtual photon polarization vectors, we adopt the form:

$$\varepsilon_{\lambda=0} = \frac{1}{\sqrt{Q^2}}(|\vec{k}|, 0, 0, \omega) \ , \qquad \varepsilon_{\lambda=\pm1} = \mp\frac{1}{\sqrt{2}}\,(0, 1, \pm i, 0) \ . \tag{80}$$

When this vectors are used in the c.m. frame, $\varepsilon_{\lambda=0}$ has to be calculated with $|\vec{k}^*|$ and ω^*. Knowing this, we can cast the transition amplitude in the form:

$$\frac{1}{4}\sum_{\lambda' s}|\mathcal{T}|^2 = \frac{1}{4}\frac{e^2}{Q^4}\sum_{\lambda\lambda'=0\pm1}\mathcal{L}_{\lambda\lambda'}\mathcal{H}_{\lambda\lambda'}\ , \tag{81}$$

with the hadronic part, that will be discussed later, as:

$$\mathcal{H}_{\lambda\lambda'} = \sum_{\lambda_1\lambda_2}\mathcal{M}_{\lambda}^{\lambda_1\lambda_2}\left(\mathcal{M}_{\lambda'}^{\lambda_1\lambda_2}\right)^{\dagger}\ . \tag{82}$$

Since the leptonic tensor is given by $l_\mu^{\lambda_1'\lambda_2'} = \overline{U}_{e_2}\,(k_2,\lambda_2')\,\gamma_\mu U_{e_1}\,(k_1,\lambda_1')$ we can cast $\mathcal{L}_{\lambda\lambda'}$ in the form

$$\begin{aligned}
\mathcal{L}_{\lambda\lambda'} &= \sum_{\lambda_1\lambda_2}(-1)^{\lambda+\lambda'}\,L_{\lambda}^{\lambda_1\lambda_2}\left(L_{\lambda'}^{\lambda_1\lambda_2}\right)^{\dagger}\ , \\
\mathcal{L}_{\lambda\lambda'} &= (-1)^{\lambda+\lambda'}\,\mathrm{Tr}\{(\not{k}_2+m_e)\ \not{\varepsilon}_{\lambda}^{*}\,(\not{k}_1+m_e)\ \not{\varepsilon}_{\lambda'}\}\ , \\
&= 4\,(-1)^{\lambda+\lambda'}\left[-\frac{1}{2}\,(\varepsilon_{\lambda}^{*}\cdot\varepsilon_{\lambda'})\,Q^2+(k_2\cdot\varepsilon_{\lambda}^{*})\,(k_1\cdot\varepsilon_{\lambda'})+(k_1\cdot\varepsilon_{\lambda}^{*})\,(k_2\cdot\varepsilon_{\lambda'})\right]\ .
\end{aligned} \tag{83}$$

Evaluating this in the lab frame gives:

$$\mathcal{L}_{\lambda\lambda'} = 8\,\epsilon_1\epsilon_2\ \cos^2\left(\frac{\theta_e}{2}\right)\widetilde{\mathcal{L}}_{\lambda\lambda'}\ , \tag{84}$$

with:

$$\widetilde{\mathcal{L}}_{0,0} = \frac{Q^2}{|\vec{k}|^2} \equiv \frac{|\vec{k}|^2}{Q^2}\nu_{L}\ , \tag{85}$$

$$\widetilde{\mathcal{L}}_{1,1} = \widetilde{\mathcal{L}}_{-1,-1} = \left[\tan^2\left(\frac{\theta_e}{2}\right)+\frac{Q^2}{2|\vec{k}|^2}\right] \equiv \nu_{T}\ , \tag{86}$$

$$\begin{aligned}
\widetilde{\mathcal{L}}_{0,1} &= \widetilde{\mathcal{L}}_{1,0} = -\widetilde{\mathcal{L}}_{0,-1} = -\widetilde{\mathcal{L}}_{-1,0} \\
&= -\left[\frac{Q^2}{2|\vec{k}|^2}\left(\frac{Q^2}{|\vec{k}|^2}+\tan^2\left(\frac{\theta_e}{2}\right)\right)\right]^{1/2} \equiv \sqrt{\frac{|\vec{k}|^2}{Q^2}}\nu_{TL}\ ,
\end{aligned} \tag{87}$$

$$\widetilde{\mathcal{L}}_{1,-1} = \widetilde{\mathcal{L}}_{-1,1} = -\frac{Q^2}{2|\vec{k}|^2} \equiv \nu_{TT}\ . \tag{88}$$

$$\tag{89}$$

Now, we can return to the unpolarized differential cross section that can be cast in a form like:

$$\begin{aligned}
\frac{d\sigma}{d\epsilon_2\,d\Omega_2\,d\Omega_K^*} &= \frac{\alpha\cos^2\left(\frac{\theta_e}{2}\right)}{\left(2\epsilon_1\sin^2\left(\frac{\theta_e}{2}\right)\right)^2}\frac{K_H}{2\pi^2}\left[\nu_L\left(\frac{W}{M_p}\right)^2\frac{d\sigma_L}{d\Omega_K^*}+\nu_T\frac{d\sigma_T}{d\Omega_K^*}\right. \\
&\left. +\nu_{TT}\frac{d\sigma_{TT}}{d\Omega_K^*}\cos\left(2\phi_K^*\right)+\nu_{TL}\left(\frac{W}{M_p}\right)\frac{d\sigma_{TL}}{d\Omega_K^*}\cos\left(\phi_K^*\right)\right]\ .
\end{aligned} \tag{90}$$

Remark that the factors (W/M_p) arise after the Lorentz transformation of the virtual photon three momentum since $|\vec{k}| = \left(\frac{W}{M_p}\right)|\vec{k}^*|$. The kaon production cross section $d\sigma_L/d\Omega^*_X$ for the longitudinal ($X = L$), transverse ($X = T$) and interference transverse-transverse ($X = TT$) and transverse-longitudinal ($X = TL$) polarizations of the virtual photon in the c.m. frame at $\phi^*_K = 0$ are given by:

$$\frac{d\sigma_L}{d\Omega^*_K} = \chi\,\frac{|\vec{k}^*|^2}{Q^2}\frac{1}{(4\pi)^2}\mathcal{H}_{0,0}\ , \tag{91}$$

$$\frac{d\sigma_T}{d\Omega^*_K} = \chi\,\frac{1}{(4\pi)^2}\left(\mathcal{H}_{1,1}+\mathcal{H}_{-1,-1}\right)\ , \tag{92}$$

$$\frac{d\sigma_{TT}}{d\Omega^*_K} = \chi\,\frac{1}{(4\pi)^2}\left(\mathcal{H}_{1,-1}+\mathcal{H}_{-1,1}\right)\ , \tag{93}$$

$$\frac{d\sigma_{TL}}{d\Omega^*_K} = \chi\sqrt{\frac{|\vec{k}^*|^2}{Q^2}}\frac{1}{(4\pi)^2}\left(\mathcal{H}_{0,1}+\mathcal{H}_{1,0}-\mathcal{H}_{-1,0}-\mathcal{H}_{0,-1}\right)\ , \tag{94}$$

where:

$$\chi = \frac{1}{16}\frac{1}{WM_p}\frac{|\vec{p}_K^{\,*}|}{K_H}\ . \tag{95}$$

It is worth remarking that the spin averaging factor 1/4 is included in the factor 1/16. So, if polarization observables are calculated, this factor has to be modified. The equivalent real photon lab energy K_H is:

$$K_H = \omega - \frac{Q^2}{2M_p} = \frac{s-M_p^2}{2M_p}\ . \tag{96}$$

In the Eq.(90) the ϕ^*_K dependence is explicit given, however, the cosines arise by their self in the $\mathcal{H}_{\lambda\lambda'}$ functions in the four vector products of the virtual photon polarization vectors and the external four momenta.
The factors $1/(4\pi)^2$ are here given in combination with the hadronic tensors. This is because the two coupling constants in every hadronic amplitude are always normalized on the value $1/\sqrt{4\pi}$ and so, in fact these factors are absorbed by the hadronic tensors.

In many references [33, 25], an other decomposition of the electroproduction cross section is maintained. The differential cross section can also be written as:

$$\frac{d\sigma}{d\epsilon_2\, d\Omega_2\, d\Omega^*_K} = \Gamma\left[\frac{d\sigma_T}{d\Omega^*_K}+\epsilon\frac{d\sigma_L}{d\Omega^*_K}+\epsilon\frac{d\sigma_{TT}}{d\Omega^*_K}\cos\left(2\phi^*_K\right)+\sqrt{\epsilon\left(1+\epsilon\right)}\frac{d\sigma_{TL}}{d\Omega^*_K}\cos\left(\phi^*_K\right)\right]\ . \tag{97}$$

Herein is

$$\Gamma = \frac{\alpha}{2\pi^2}\frac{\epsilon_2}{\epsilon_1}\frac{K_H}{Q^2}\frac{1}{1-\epsilon}\ , \tag{98}$$

and ϵ the degree of transverse polarization of the virtual photon, given by:

$$\epsilon = \left(1+\frac{2|\vec{k}|^2}{Q^2}\tan^2\frac{\theta_e}{2}\right)^{-1}\ . \tag{99}$$

ϵ_L can be defined as:

$$\epsilon_L = \frac{Q^2}{|\vec{k}|^2}\,\epsilon\ , \tag{100}$$
$$= \frac{Q^2}{\omega^2}\,\epsilon\ , \tag{101}$$
$$= \frac{Q^2}{\omega^{*2}}\,\epsilon\ . \tag{102}$$

Note that different conventions are used in literature! ϵ is invariant under collinear transformations and can be evaluated with $\vec{k}$ and θ_e in the lab or c.m. frame. ϵ_L does not fulfill this invariance and is frame dependent.

The virtual photon cross sections are now defined as:

$$\frac{d\sigma_L}{d\Omega_K^*} = 2\,\chi\,\frac{1}{(4\pi)^2}\mathcal{H}_{0,0}\ , \tag{103}$$
$$\frac{d\sigma_T}{d\Omega_K^*} = \chi\,\frac{1}{(4\pi)^2}\left(\mathcal{H}_{1,1}+\mathcal{H}_{-1,-1}\right)\ , \tag{104}$$
$$\frac{d\sigma_{TT}}{d\Omega_K^*} = -\,\chi\,\frac{1}{(4\pi)^2}\left(\mathcal{H}_{1,-1}+\mathcal{H}_{-1,1}\right)\ , \tag{105}$$
$$\frac{d\sigma_{TL}}{d\Omega_K^*} = -\,\chi\,\frac{1}{(4\pi)^2}\left(\mathcal{H}_{0,1}+\mathcal{H}_{1,0}-\mathcal{H}_{-1,0}-\mathcal{H}_{0,-1}\right)\ , \tag{106}$$

where χ is defined as in Eq.(95):

$$\chi = \frac{1}{16}\frac{1}{W^2}\frac{|\vec{p}_K^{\,*}|}{K_H}\frac{|\vec{k}|}{|\vec{k}^*|}\ , \tag{107}$$

The hadronic part occurring in the expansion (81) can cast in a trace calculation:

$$\begin{aligned}
\mathcal{H}_{\lambda\lambda'} &= \sum_{\lambda_1\lambda_2}\mathcal{M}_{\lambda}^{\lambda_1\lambda_2}\left(\mathcal{M}_{\lambda'}^{\lambda_1\lambda_2}\right)^{\dagger}\ ,\\
&= \sum_{\lambda_1\lambda_2}\overline{U}_Y^{\lambda_2}(p_Y)\,T^{\mu}\varepsilon_{\mu}^{\lambda}U_p^{\lambda_1}(p)\,\overline{U}_p^{\lambda_1}(p)\,\overline{T}^{\nu}\varepsilon_{\nu}^{\lambda'*}U_Y^{\lambda_2}(p_Y)\ ,\\
&= \mathrm{Tr}\left\{(\not{p}_Y+M_Y)\,T^{\mu}\varepsilon_{\mu}^{\lambda}(\not{p}+M_p)\,\overline{T}^{\nu}\varepsilon_{\nu}^{\lambda'*}\right\}\ .
\end{aligned} \tag{108}$$

The result of this trace calculation is a function of:

$$(p\cdot k) = \omega^*\sqrt{M_p^2+|\vec{k}^*|^2}\ +\ |\vec{k}^*|^2\ , \tag{109}$$
$$(p_K\cdot k) = \omega^*\sqrt{M_K^2+|\vec{p}_K^{\,*}|^2}\ -\ |\vec{k}^*|\,|\vec{p}_K^{\,*}|\cos\theta_K^*\ , \tag{110}$$
$$(p_Y\cdot k) = \omega^*\sqrt{M_Y^2+|\vec{p}_K^{\,*}|^2}\ +\ |\vec{k}^*|\,|\vec{p}_K^{\,*}|\cos\theta_K^*\ , \tag{111}$$

$$
\begin{aligned}
(p\cdot p_K) &= \sqrt{M_p^2+|\vec{k}^*|^2}\sqrt{M_K^2+|\vec{p}_K^{\,*}|^2}\; + \;|\vec{k}^*|\,|\vec{p}_K^{\,*}|\cos\theta_K^* \;, && (112)\\
(p\cdot p_Y) &= \sqrt{M_p^2+|\vec{k}^*|^2}\sqrt{M_Y^2+|\vec{p}_K^{\,*}|^2}\; - \;|\vec{k}^*|\,|\vec{p}_K^{\,*}|\cos\theta_K^* \;, && (113)\\
(p_Y\cdot p_K) &= \sqrt{M_Y^2+|\vec{p}_K^{\,*}|^2}\sqrt{M_K^2+|\vec{p}_K^{\,*}|^2}+|\vec{p}_K^{\,*}|^2 \;, && (114)\\
(\varepsilon^\lambda\cdot k) &= 0\;, && (115)\\
(\varepsilon^\lambda\cdot p) &= \frac{|\vec{k}^*|}{\sqrt{Q^2}}\left(\sqrt{M_p^2+|\vec{k}^*|^2}+w^*\right)\,\delta_{\lambda,0}\;, && (116)\\
(\varepsilon^\lambda\cdot p_K) &= \begin{cases}\frac{1}{\sqrt{Q^2}}\left(|\vec{k}^*|\sqrt{M_K^2+|\vec{p}_K^{\,*}|^2}-\omega^*|\vec{p}_K^{\,*}|\cos\theta_K^*\right) & ;\ \lambda=0\\ \frac{\lambda}{\sqrt{2}}\,|\vec{p}_K^{\,*}|\sin\theta_K^*\,(\cos\phi_K^*+i\lambda\sin\phi_K^*) & ;\ \lambda=\pm 1\end{cases} && (117)\\
(\varepsilon^\lambda\cdot p_Y) &= \begin{cases}\frac{1}{\sqrt{Q^2}}\left(|\vec{k}^*|\sqrt{M_Y^2+|\vec{p}_K^{\,*}|^2}+\omega^*|\vec{p}_K^{\,*}|\cos\theta_K^*\right) & ;\lambda=0\\ \frac{-\lambda}{\sqrt{2}}\,|\vec{p}_K^{\,*}|\sin\theta_K^*\,(\cos\phi_K^*+i\lambda\sin\phi_K^*) & ;\lambda=\pm 1\end{cases} && (118)\\
(\varepsilon^\lambda\cdot\varepsilon^{\lambda'*}) &= (-1)^\lambda\,\delta_{\lambda,\lambda'}\;, && (119)
\end{aligned}
$$

Note that the hadronic tensors are calculated for the $\phi_K^*=0$ situation. Consequently, there is no explicit ϕ_K^* dependence in the calculation of the four vector products. Inspecting the equations (97, 110 - 119), it becomes clear that the electroproduction cross section is function of four independent variables. Possible choices for this independent set are $(\omega, |\vec{k}|, \theta_e, \theta_K, \phi_K)$ or $(\epsilon, s, t, Q^2, \phi_K)$. The transformation relations between this two set are:

$$
\begin{aligned}
\epsilon &= \left(1+2\frac{|\vec{k}|^2}{Q^2}\tan^2\frac{\theta_e}{2}\right)^{-1}\;, && (120)\\
Q^2 &= |\vec{k}^*|^2-\omega^{*2} \;=\; |\vec{k}|^2-\omega^2\;, && (121)\\
s &= M_p^2-Q^2+2\left(E_p^*\omega^*+|\vec{k}^*|^2\right) \;=\; M_p^2-Q^2+2M_p\omega\;, && (122)\\
t &= M_K^2-Q^2-2\left(E_K^*\omega^*-|\vec{k}^*||\vec{p}_K^{\,*}|\cos\theta_K^*\right)\;, && (123)
\end{aligned}
$$

and the inverse relations are given by:

$$
\begin{aligned}
\theta_e &= 2\arctan\left(\sqrt{\frac{Q^2}{2|\vec{k}|^2}\left(\frac{1-\epsilon}{\epsilon}\right)}\right)\;, && (124)\\
\omega &= \frac{s+Q^2-M_p^2}{2M_p}\;, && (125)\\
|\vec{k}| &= \sqrt{Q^2+\omega^2}\;, && (126)\\
\cos\theta_K^* &= \frac{t+Q^2+2E_K^*\omega^*-M_K^2}{2|\vec{p}_K^{\,*}||\vec{k}^*|}\;. && (127)
\end{aligned}
$$

Beside these sets of independent variables, we also need some depended quantities like in Eq.(98), the incoming and outgoing electron energies:

$$\epsilon_1 = \frac{1}{2}\left(\sqrt{\omega^2 + \frac{2Q^2}{1-\cos\theta_e}} + \omega\right) , \tag{128}$$

$$\epsilon_2 = \frac{1}{2}\left(\sqrt{\omega^2 + \frac{2Q^2}{1-\cos\theta_e}} - \omega\right) . \tag{129}$$

$$\tag{130}$$

When the beam energy is known, the electron scattering angle is defined by:

$$\cos\theta_e = 1 - \frac{Q^2}{2\epsilon_1\left(\epsilon_1 - \omega\right)} . \tag{131}$$

2.2.2 Polarized electron beam

So far, the expressions for electroproduction were derived for unpolarized particles both in the initial as well as in the final state. When polarized electrons are used in the experiment, two additional terms enter the expression for the cross section:

$$\frac{d\sigma}{d\epsilon_2 d\Omega_2 d\Omega_K} = \left.\frac{d\sigma}{d\epsilon_2 d\Omega_2 d\Omega_K}\right|_{unpol} + h\,\Gamma\left[\sqrt{1-\epsilon^2}\frac{d\sigma_{TT'}}{d\Omega_K} + \sqrt{\epsilon\left(\epsilon-1\right)}\frac{d\sigma_{TL'}}{d\Omega_K}\sin\left(\phi_K\right)\right] . \tag{132}$$

Herein, h is the helicity of the incident electron. Further, the two additional virtual photon cross sections are defined as:

$$\frac{d\sigma_{TT'}}{d\Omega_K} = \chi\,\frac{1}{(4\pi)^2}\left(\mathcal{H}_{1,1} - \mathcal{H}_{-1,-1}\right) , \tag{133}$$

$$\frac{d\sigma_{TL'}}{d\Omega_K} = -\chi\,\frac{i}{(4\pi)^2}\left(\mathcal{H}_{1,0} - \mathcal{H}_{0,1} - \mathcal{H}_{-1,0} + \mathcal{H}_{0,-1}\right) , \tag{134}$$

where the hadronic tensors are as in Eq. (108). Because of the structure of the hadron current, the TT' term is identical to zero if no baryon polarizations are involved in the process.

2.2.3 Polarized Hadrons

In the case of polarized hadrons, the cross section is commonly expressed in terms of response functions:

$$\begin{aligned}\frac{d\sigma}{d\epsilon_2 d\Omega_2 d\Omega_K^*} = \Gamma\,\rho\,S_\alpha\,S_\beta\,\Big[& R_T^{\alpha\beta} + \epsilon R_L^{\alpha\beta} + \epsilon\left({}^cR_{TT}^{\alpha\beta}\cos\left(2\phi_K^*\right) +{}^s R_{TT}^{\alpha\beta}\sin\left(2\phi_K^*\right)\right) \\ &+\sqrt{\epsilon\left(1+\epsilon\right)}\left({}^cR_{TL}^{\alpha\beta}\cos\left(\phi_K^*\right) +{}^s R_{TL}^{\alpha\beta}\sin\left(\phi_K^*\right)\right) + h\sqrt{1-\epsilon^2}R_{TT'}^{\alpha\beta} \\ &+h\sqrt{\epsilon\left(1-\epsilon\right)}\left({}^cR_{TL'}^{\alpha\beta}\cos\left(\phi_K^*\right) +{}^s R_{TL'}^{\alpha\beta}\sin\left(\phi_K^*\right)\right)\Big] .\end{aligned} \tag{135}$$

Here, $\rho = |\vec{p}_K| / \left|\vec{k}\right|$. Further, the selection of the spin quantization axis is performed by the target and recoil spin vectors $S_\alpha = (1, S_x, S_y, S_z)$ and $S_\beta = (1, S_{x'}, S_{y'}, S_{z'})$, respectively. The (x, y, z) and (x', y', z')-frame are as given in Fig. 2.2.1. The response functions are defined in terms of the hadronic tensors by the following relations:

$$\begin{aligned}
R_T^{\alpha\beta} &= \rho^{-1}\,\chi\,\frac{1}{(4\pi)^2}\,(H_{1,1} + H_{-1,-1}) \\
R_L^{\alpha\beta} &= \rho^{-1}\,2\,\chi\,\frac{1}{(4\pi)^2} H_{0,0} \\
{}^{c}R_{TT}^{\alpha\beta} &= -\rho^{-1}\,\chi\,\frac{1}{(4\pi)^2}\,(H_{1,-1} + H_{-1,1}) \\
{}^{s}R_{TT}^{\alpha\beta} &= -\rho^{-1}\,\chi\,\frac{i}{(4\pi)^2}\,(H_{1,-1} - H_{-1,1}) \\
{}^{c}R_{TL}^{\alpha\beta} &= -\rho^{-1}\,\chi\,\frac{1}{(4\pi)^2}\,(H_{1,0} + H_{0,1} - H_{-1,0} - H_{0,-1}) \\
{}^{s}R_{TL}^{\alpha\beta} &= -\rho^{-1}\,\chi\,\frac{i}{(4\pi)^2}\,(H_{1,0} - H_{0,1} + H_{-1,0} - H_{0,-1}) \\
R_{TT'}^{\alpha\beta} &= \rho^{-1}\,\chi\,\frac{1}{(4\pi)^2}\,(H_{1,1} - H_{-1,-1}) \\
{}^{c}R_{TL'}^{\alpha\beta} &= -\rho^{-1}\,\chi\,\frac{1}{(4\pi)^2}\,(H_{1,0} + H_{0,1} + H_{-1,0} + H_{0,-1}) \\
{}^{s}R_{TL'}^{\alpha\beta} &= -\rho^{-1}\,\chi\,\frac{i}{(4\pi)^2}\,(H_{1,0} - H_{0,1} - H_{-1,0} + H_{0,-1}) \qquad (136)
\end{aligned}$$

It is important to stress that all hadron tensors in the expressions given above are defined for $\phi_K^* = 0$. If the ϕ_K^*-dependence is explicitly taken into account in the calculation of the four vector products, the ${}^{s}R_X^{\alpha\beta}$ terms follows immediately from the ${}^{c}R_X^{\alpha\beta}$.

2.2.4 Response functions and transferred recoil polarization

The observed polarizations can be expressed in terms of the response functions. Those polarization can be measured in the (x, y, z), (n, t, l) or (x', y', z') frame. For a full overview over the polarizations in the different frames, we refer to the notes of Daniel Carman.

In this section, we only discuss the transfered recoil polarization. Analogue equations exist for the other polarization observables. The transfered polarization in the (x', y', z') reference frame in given by:

$$\begin{aligned}
P'_{x'} &= c^- \, {}^{c}R_{TL'}^{x'0} \cos\phi + c^0 R_{TT'}^{x'0} \,, \\
P'_{y'} &= c^- \, {}^{s}R_{TL'}^{y'0} \sin\phi \,, \\
P'_{z'} &= c^- \, {}^{c}R_{TL'}^{z'0} \cos\phi + c^0 R_{TT'}^{z'0} \,. \qquad (137)
\end{aligned}$$

Here, $c^- = \sqrt{\epsilon(1-\epsilon)}/\sigma_0$ and $c^0 = \sqrt{1-\epsilon^2}/\sigma_0$. Note that the response functions are defined in the (x', y', z') frame. When the polarization is measured in the (x, y, z) frame, the same set of equations holds if also the response functions are expressed in the (x, y, z) frame. The polarizations in the (x, y, z) frame in terms of the response functions in the (x', y', z') become:

$$
\begin{aligned}
P'_x &= c^- \, {}^cR^{x'0}_{TL'} \cos^2\phi \cos\theta + c^0 R^{x'0}_{TT'} \cos\phi \cos\theta - c^- \, {}^sR^{y'0}_{TL'} \sin^2\phi + \\
&\quad c^- \, {}^cR^{z'0}_{TL'} \cos^2\phi \sin\theta + c^0 R^{z'0}_{TT'} \cos\phi \sin\theta \ , \\
P'_y &= c^- \, {}^cR^{x'0}_{TL'} \cos\phi \sin\phi \cos\theta + c^0 R^{x'0}_{TT'} \sin\phi \cos\theta + c^- \, {}^sR^{y'0}_{TL'} \sin\phi \cos\phi + \\
&\quad c^- \, {}^cR^{z'0}_{TL'} \cos\phi \sin\phi \sin\theta + c^0 R^{z'0}_{TT'} \sin\phi \sin\theta \ , \\
P'_z &= -c^- \, {}^cR^{x'0}_{TL'} \cos\phi \sin\theta - c^0 R^{x'0}_{TT'} \sin\theta + \\
&\quad c^- \, {}^cR^{z'0}_{TL'} \cos\phi \cos\theta + c^0 R^{z'0}_{TT'} \cos\theta \ .
\end{aligned}
\tag{138}
$$

From the above set of equations, we can construct the transformation relations between the response functions in the (x, y, z) and the (x', y', z') frame:

$$
\begin{aligned}
{}^cR^{x0}_{TL'} &= {}^cR^{x'0}_{TL'} \cos\phi \cos\theta - {}^sR^{y'0}_{TL'} \tan\phi \sin\phi + {}^cR^{z'0}_{TL'} \cos\phi \sin\theta \\
R^{x0}_{TT'} &= R^{x'0}_{TT'} \cos\phi \cos\theta + R^{z'0}_{TT'} \cos\phi \sin\theta \\
{}^sR^{y0}_{TL'} &= {}^cR^{x'0}_{TL'} \cos\phi \cos\theta + {}^sR^{y'0}_{TL'} \cos\phi + {}^cR^{z'0}_{TL'} \cos\phi \sin\theta \\
R^{y0}_{TT'} &= R^{x'0}_{TT'} \sin\phi \cos\theta + R^{z'0}_{TT'} \sin\phi \sin\theta \\
{}^cR^{z0}_{TL'} &= -\,{}^cR^{x'0}_{TL'} \sin\theta + {}^cR^{z'0}_{TL'} \cos\theta \\
R^{z0}_{TT'} &= -R^{x'0}_{TT'} \sin\theta + R^{z'0}_{TT'} \cos\theta
\end{aligned}
\tag{139}
$$

2.3 Radiative Kaon Capture

On the basis of crossing symmetry, one can connect the reaction dynamics of the radiative kaon capture process:

$$K^- + p \to \gamma + Y \, ; \qquad Y = \Lambda, \Sigma^0 \tag{140}$$

to the reaction dynamics of the kaon photoproduction process. Technically speaking, this connection between the Feynman amplitudes reads:

$$\mathcal{M}^{K^- p \to \gamma Y}(p, k, p_K, p_Y) = \mathcal{M}^{\gamma p \to K^+ Y}(p, -k, -p_K, p_Y) \ . \tag{141}$$

The interchange of the signs of the photon and kaon four momenta ensures that s- and u-channel diagrams are converted and that the operational structure of the vertex functions is handled correctly.

So far, there are no data available for the reaction (140). The only available measurement for this type of reaction is a branching ration of the radiative decay of *stopped* kaons:

$$R \equiv \frac{\Gamma(K^- p \to \gamma Y)}{\Gamma(K^- p \to \text{all})} \ . \tag{142}$$

On the basis of the work of Bardeen and Torigoe [4], the decay width $\Gamma(K^-p \to \text{all})$ can be related to the K^-p pseudo-potential:

$$\Gamma\left(K^-p \to \text{all}\right) = 2W_{K^-p} \left|\Phi(0)\right|^2 \ . \tag{143}$$

The pseudo-potential is determined by Burkhardt *et al.* [9] on the basis of the K^-p scattering amplitude:

$$W_{K^-p} = 560 \pm 135 \ \text{MeV fm}^3 \ . \tag{144}$$

Further in Eq. (143), $\Phi(0)$ is the kaon atomic wave function at the origin. It is shown by Leon and Bethe [26] that radiative capture process takes place from an S-state with a probability more than 99%.

In order to obtain an expression for the decay width $\Gamma(K^-p \to \gamma Y)$, we start from the general expression of Peskin and Schoeder [35]:

$$\Gamma = \frac{1}{2E_p}\frac{1}{2E_K}\int \frac{d^3\vec{p}_Y}{(2\pi)^3}\frac{1}{2E_Y}\frac{d^3\vec{k}}{(2\pi)^3}\frac{1}{2\omega}(2\pi)^4\delta^{(4)}\left(p + p_K - p_Y - k\right)\overline{\sum_{\text{spins}}}\left|\mathcal{M}^{K^-p\to\gamma Y}\right|^2 \ , \tag{145}$$

Note that this formula contains the phase space factors $\frac{1}{2E_p}\frac{1}{2E_K}$ instead of $\frac{1}{2E_R}$ which is appropriate for the decay of an unstable resonance particle R. These two new phase space factors are motivated by the fact that we start with to particles in the initial state (a K^- and a p) which are connected to each other in an S-state by the wave function $\Phi(0)$. This wave function has to be taken into account in the transition amplitude:

$$M^{K^-p\to\gamma Y} = \overline{u}_Y(p_Y)\,T^{\mu}\varepsilon_{\mu}u_p(p)\,\Phi(0) \ . \tag{146}$$

Simplifying Eq. (145) with the knowledge of Eq. (146) gives rise to the expression:

$$\Gamma\left(K^-p \to \gamma Y\right) = \frac{1}{16\pi}\frac{\omega}{M_pM_K\left(M_p + M_K\right)}\left|\Phi(0)\right|^2\frac{1}{2}\left|\mathcal{M}^{\gamma p\to K^+Y}\left(p, -k, -p_K, p_Y\right)\right|^2 \tag{147}$$

Combining the above expressions of the decay widths results in a branching ratio:

$$\begin{aligned} R &= \frac{\Gamma(K^-p \to \gamma Y)}{\Gamma(K^-p \to \text{all})} \\ &= \frac{\pi}{2W_{K^-p}}\frac{\omega}{M_pM_K\left(M_p + M_K\right)}\frac{1}{(4\pi)^2}\frac{1}{2}\left|\mathcal{M}^{\gamma p\to K^+Y}\left(p, -k, -p_K, p_Y\right)\right|^2 \end{aligned} \tag{148}$$

2.4 Trace Calculations: Complexity Problem

When performing trace calculations, the result is in principle a complex number, containing a real and imaginary part. However, in many cases, it turns out that the result is either pure real or pure imaginary. Here, we give a short summary of this topic. This is extremely important in `strangecalc` since the code does not work with complex numbers and all the imaginary numbers are taken into account analytically.

The general response function is expressed in terms of hadronic tensors $H_{\lambda,\lambda'}$. Those hadronic tensors contain a trace part and a part from the propagators:

$$R_X = \sum_{\lambda,\lambda'} [\mathcal{P}_R + i\,\mathcal{P}_I]\,[\mathcal{T}_R + i\,\mathcal{T}_I] \pm [\mathcal{P}_R - i\,\mathcal{P}_I]\,[\mathcal{T}_R - i\,\mathcal{T}_I] \ , \tag{149}$$

where $\mathcal{P}_{R,I}$ is the real/imaginary part of the propagator part and and $\mathcal{T}_{R,I}$ is the real/imaginary part of the trace result.

If $\lambda = \lambda'$, the complex conjugated part occurs in the same hadronic tensor $H_{\lambda,\lambda'}$ and the '+' has to be taken in Eq. (149). This results in:

$$R_X = \sum_{\lambda,\lambda'} 2\,[\mathcal{P}_R\mathcal{T}_R - \mathcal{P}_I\mathcal{T}_I] \ , \qquad \text{if:} \quad + \ . \tag{150}$$

If $\lambda \neq \lambda'$, the complex conjugated part of a certain term occurs in an other hadronic tensor. It then depends on the relative signs, as for example given in Eqs. (136), if the real or imaginary part survives:

$$R_X \sim \begin{cases} \sum_{\lambda,\lambda'} 2\,[\mathcal{P}_R\mathcal{T}_R - \mathcal{P}_I\mathcal{T}_I] \ , & \text{if:} \quad + \ , \\ i\, \sum_{\lambda,\lambda'} 2i\ [\mathcal{P}_R\mathcal{T}_I + \mathcal{P}_I\mathcal{T}_R] \ , & \text{if:} \quad - \ , \end{cases} \tag{151}$$

Note that the definition of the response functions as given in Eqs. (136) contains an additional factor 'i' if the imaginary part of the amplitude is selected. The two imaginary factors give and extra factor -1.

2.4.1 Photoproduction

In the real photoproduction processes, regardless the polarization of the external particles, the cross section is determined by the hadronic tensors $H_{1,1}$ and $H_{-1,-1}$. As such, only the overall real part survives as given in Eq. (150).

- **Unpolarized Reaction:** The external four vectors are the only quantities that enter the trace calculation. They are real which result in a real trace: $\mathcal{T}_I = 0$. The response function becomes:

$$R_X \sim \mathcal{P}_R\mathcal{T}_R \ . \tag{152}$$

- **Single Polarization:**
  - **Beam Polarization:** There is no asymmetric part for circular polarized photons. Only linear polarized photons give rise to an asymmetric part. The linear polarized photons are real, the trace is real and:

$$R_X \sim \mathcal{P}_R\mathcal{T}_R \ . \tag{153}$$

– **Baryon Polarization:** There is only an asymmetry when one of the baryons is polarized along the $\vec{y}$-axis. The additional γ_5 in the trace gives rise to a determinant according to Eq. (179) with a factor 'i' in front. With a polarized baryon, the trace is imaginary ($\mathcal{T}_R$ = 0) and:

$$R_X \sim \mathcal{P}_I \mathcal{T}_I \ . \tag{154}$$

- **Double Polarization:** The polarized baryon gives rise to a factor i due to the γ_5 in the trace. When the photon beam is **linearly polarized**, the polarization vectors are real and the trace is imaginary ($\mathcal{T}_R$ = 0). When the photon beam is **circularly polarized**, there is an imaginary $\vec{y}$-component in the polarization vector. The result is a real trace ($\mathcal{T}_I$ = 0). Summarizing:

$$R_X \sim \mathcal{P}_R \mathcal{T}_R \quad \text{circularly pol. photons} \ , \tag{155}$$
$$R_X \sim \mathcal{P}_I \mathcal{T}_I \quad \text{linearly pol. photons} \ . \tag{156}$$

2.4.2 Electroproduction

For electroproduction, the things are slightly more complicated. The relative sign between the various hadronic tensor is not always '+' anymore. As a consequence, both the overall real and/or imaginary part can contribute to the response functions as given in Eq. (151).

- **Unpolarized or Polarized Beam:** The virtual photon polarization vector can have a imaginary $\vec{y}$ component (if $\lambda = \pm$) but since all other external four vectors have not such a component, the trace is real. Remark that this is only true if ϕ_{K^*} = 0. Otherwise, the external four vectors have a $\vec{y}$ component which has to be combined with the $\vec{y}$ component of the ε vector. Since the ϕ_{K^*} dependence is already extracted from the structure functions, we can calculate them at ϕ_{K^*} = 0. For unpolarized baryons in the reaction, the structure functions T, L, TT and TL are always composed by a relative '+' sign between the hadronic tensors. As such the overall real part contributes here:

$$R_X \sim \mathcal{P}_R \mathcal{T}_R \ . \tag{157}$$

- **Baryon Polarization:** This is the most complicated situation. When one of the baryons is polarized the trace contains both real and imaginary parts. The four vector products that generate an imaginary result are:

$$\begin{array}{cl} (\varepsilon^{\pm} \cdot n) & \text{if: } \vec{n} = \vec{y} \\ \det(\varepsilon^{\pm}\ \varepsilon^{\pm *}\ n\ a) & \text{independent of } n \\ \det(\varepsilon^{\pm}\ n\ a\ b) & \text{if: } \vec{n} \neq \vec{y} \end{array} \tag{158}$$

Here, a and b are four vectors with a vanishing $\vec{y}$ component. Note that this can also be a ε^0 vector. The response function becomes now a mixed result:

$$R_X \sim \begin{cases} \sum_{\lambda,\lambda'} 2\left[\mathcal{P}_R \mathcal{T}_R - \mathcal{P}_I \mathcal{T}_I\right] \ , & \text{if: } + \ , \\ -\sum_{\lambda,\lambda'} 2\left[\mathcal{P}_R \mathcal{T}_I + \mathcal{P}_I \mathcal{T}_R\right] \ , & \text{if: } - \ , \end{cases} \tag{159}$$

3 The Interaction Lagrangians

The expression for the amplitude $\mathcal{M}_{\lambda}^{\lambda_1\lambda_2}$ which occurs in the cross section is constructed on field theoretical grounds. We start from an effective Lagrangian formalism in which the leading contributions to the amplitude at the tree level are taken in to account. This includes the nucleon and hyperon Born terms and the leading t-channel vector meson exchanges as non-resonant pieces. In addition, we consider resonant contributions in the s- and u-channels. The different diagrams are showed in Fig.(4). The possible particles and resonances with their properties that could play a role in the production process are listed in Table 11.
In order to work in a completely consistent way, we start at the level of the interaction Lagrangians and the propagators of the intermediate (non)-resonant particles.

The electromagnetic interaction vertices are given by:

3.1 Born Terms

The electromagnetic vertices are described by:

$$\mathcal{L}_{\gamma pp} = -e\overline{N}\gamma_\mu N A^\mu + \frac{e\kappa_p}{4M_p}\overline{N}\sigma_{\mu\nu}NF^{\mu\nu} , \tag{160}$$

$$\mathcal{L}_{\gamma YY} = -e\overline{Y}\gamma_\mu Y A^\mu + \frac{e\kappa_Y}{4M_p}\overline{Y}\sigma_{\mu\nu}YF^{\mu\nu} , \tag{161}$$

$$\mathcal{L}_{\gamma YY'} = \frac{e\kappa_{YY'}}{4M_p}\overline{Y'}\sigma_{\mu\nu}YF^{\mu\nu} + \text{h.c.} , \tag{162}$$

$$\mathcal{L}_{\gamma KK} = -ie\left(K^\dagger\partial_\mu K - K\partial_\mu K^\dagger\right)A^\mu . \tag{163}$$

Remark that the electric term in Eq. (161) only occurs in the case that $Y \equiv \Sigma^+$. If Y is and neutral hyperon, there is only magnetic coupling.
The hadronic vertex reads:

$$\mathcal{L}_{KYp} = g_{KYp}\left(-i\zeta K^\dagger\overline{Y}\gamma_5 N + (1-\zeta)\frac{1}{M_p+M_Y}\left(\partial^\mu K^\dagger\right)\overline{Y}\gamma_\mu\gamma_5 N\right) + \text{h.c.} . \tag{164}$$

In Eq.(164) is ζ a parameter to distinguish between a pseudo-scalar and a pseudo-vector coupling. In this work, we will choose $\zeta = 1$ and work with a pseudo-scalar coupling. As long as on-shell particles are concerned, both couplings produce the same results. Eq.(160) is the well known photon-nucleon coupling and can be found in every standard work. Eq.(161)-(163) are from [1] and Eq.(164) is taken from [6].

3.2 Vector Meson Exchange

The electromagnetic vertex is described by:

$$\mathcal{L}_{\gamma KV}(P=-) = \frac{eg_{\gamma KV}}{4M}\epsilon^{\mu\nu\lambda\sigma}F_{\mu\nu}V_{\lambda\sigma}K\,, \tag{165}$$

$$\mathcal{L}_{\gamma KV}(P=+) = i\frac{eg_{\gamma KV}}{M}\left(\partial_\mu A_\nu\partial^\mu V^\nu - \partial_\mu A_\nu\partial^\nu V^\mu\right)K\,. \tag{166}$$

The hadronic interaction reads:

$$\mathcal{L}_{VYp} = -g^{v}_{VYp}\overline{Y}\Gamma_\mu N V^\mu + \frac{g^{t}_{VYp}}{2\left(M_Y+M_p\right)}\overline{Y}\sigma_{\mu\nu}V^{\mu\nu}\Gamma N + \text{h.c.}\,. \tag{167}$$

These equations are found in [15] and [6]. The mass M is arbitrary chosen as 1.0 GeV.

3.3 Spin 1/2 Resonance Exchange

$$\mathcal{L}_{\gamma p R_{1/2}} = \frac{e\kappa_{pR}}{4M_p}\left(\overline{R}\Gamma_{\mu\nu}N \pm \overline{N}\Gamma_{\mu\nu}R\right)F^{\mu\nu}\,, \tag{168}$$

$$\mathcal{L}^{PS}_{KYR_{1/2}} = -ig_{KYR}\left(K^\dagger\overline{Y}\Gamma R \pm \overline{R}\Gamma Y K\right)\,, \tag{169}$$

$$\mathcal{L}^{PV}_{KYR_{1/2}} = \frac{f_{KYR}}{M_K}\left(\left(\partial^\mu K^\dagger\right)\overline{Y}\Gamma_\mu R + \overline{R}\Gamma_\mu Y\left(\partial^\mu K\right)\right)\,. \tag{170}$$

These equations are found in [15], [6] and [16], although in [15] and [16] there is an extra sign for odd parity resonances in the hadronic Lagrangians of Eq. (169) and (170).

3.4 Spin 3/2 Resonance Exchange

$$\mathcal{L}^{1}_{\gamma p R_{3/2}} = i\frac{e\kappa^{(1)}_{pR}}{2M_p}\left(\overline{R}^\mu\theta_{\mu\nu}(Y)\Gamma_\lambda N - \overline{N}\Gamma_\lambda\theta_{\nu\mu}(Y)R^\mu\right)F^{\lambda\nu}\,, \tag{171}$$

$$\mathcal{L}^{2}_{\gamma p R_{3/2}} = -\frac{e\kappa^{(2)}_{pR}}{4M_p^2}\left(\overline{R}^\mu\theta_{\mu\nu}(X)\Gamma\left(\partial_\lambda N\right) \mp \left(\partial_\lambda\overline{N}\right)\Gamma\theta_{\nu\mu}(X)R^\mu\right)F^{\nu\lambda}\,, \tag{172}$$

$$\mathcal{L}_{KYR_{3/2}} = \frac{f_{KYR}}{M_K}\left(\overline{R}^\mu\theta_{\mu\nu}(Z)\Gamma' Y\left(\partial^\nu K\right) \pm \left(\partial^\nu K^\dagger\right)\overline{Y}\Gamma'\theta_{\nu\mu}(Z)R^\mu\right)\,. \tag{173}$$

These equations are given in [30],[6] and [11]. In [15] and [16], and extra sign is introduced for the electromagnetic interaction Lagrangians in Eq.(171) and (172).

3.5 Notation

In this work, we follow the notation of Ref.[35].
In the previous Lagrangians is:

$$F^{\mu\nu} = \partial^\nu A^\mu - \partial^\mu A^\nu, \tag{174}$$
$$V^{\mu\nu} = \partial^\nu V^\mu - \partial^\mu V^\nu, \tag{175}$$

where A^μ is the photon field and V^μ the vector meson field. For the derivatives, we take the convention that

$$\begin{aligned} \partial_\mu P &= (-i)\, p_\mu P \\ \partial_\mu P^\dagger &= (i)\, p_\mu P^\dagger \end{aligned} \tag{176}$$

Were P is an incoming scalar, vector or spinor field, $P^\dagger$ is the outgoing component and p_μ is its momentum.

It is common known that $\sigma^{\mu\nu} = \frac{i}{2}[\gamma^\mu, \gamma^\nu]$. Consequently, $i\sigma^{\mu\nu} = (\gamma^\nu\gamma^\mu - g^{\mu\nu})$ and $\sigma^{\mu\nu\dagger} = \gamma^0\sigma^{\mu\nu}\gamma^0$.

The complete antisymmetric tensor is defined as:

$$\epsilon^{\mu\nu\alpha\beta} = \begin{cases} +1 & \text{for an even permutation of 0,1,2,3} \\ -1 & \text{for an odd permutation of 0,1,2,3} \\ 0 & \text{if two or more indices are the same} \end{cases} \tag{177}$$

Remark that this is according to Ref.[35]. In Ref.[8] this is the definition of $\epsilon_{\mu\nu\alpha\beta}$. This causes an additional sign in the formulas. We use here the contra variant definition since this is used in the definition of the vector meson interaction Lagrangian in Eq.(166).
Knowing that:

$$\gamma_5 \equiv \gamma^5 = i\gamma^0\gamma^1\gamma^2\gamma^3 \ , \tag{178}$$

there holds:

$$\gamma^5 = -\frac{i}{4!}\epsilon^{\mu\nu\alpha\beta}\gamma_\mu\gamma_\nu\gamma_\alpha\gamma_\beta \ . \tag{179}$$

One can also prove that:

$$-i\epsilon^{\mu\nu\alpha\beta}\gamma^5\gamma_\beta = \gamma^{\mu\nu\alpha} \equiv \gamma^{[\mu}\gamma^\nu\gamma^{\alpha]} \ , \tag{180}$$
$$= \gamma^\mu\gamma^\nu\gamma^\alpha - g^{\mu\nu}\gamma^\alpha - g^{\nu\alpha}\gamma^\mu + g^{\mu\alpha}\gamma^\nu \ , \tag{181}$$

and:

$$-i\epsilon^{\mu\nu\alpha\beta}\gamma^5 = \gamma^{\mu\nu\alpha\beta} \equiv \gamma^{[\mu}\gamma^\nu\gamma^\alpha\gamma^{\beta]} \ . \tag{182}$$

Its hermitian conjugated form is:

$$\left(i\epsilon^{\mu\nu\lambda\sigma}\right)^{\dagger} = -\gamma^{0}\left(i\epsilon^{\mu\nu\lambda\sigma}\right)\gamma^{0} \ . \tag{183}$$

The Γ-functions are depending on the parity of the resonance and defined as:

$$\Gamma = \begin{cases} 1 & ;odd \\ \gamma_5 & ;even \end{cases} \tag{184}$$

$$\Gamma' = \begin{cases} \gamma_5 & ;odd \\ 1 & ;even \end{cases} \tag{185}$$

$$\Gamma_{\mu} = \begin{cases} \gamma_{\mu} & ;odd \\ \gamma_{\mu}\gamma_5 & ;even \end{cases} \tag{186}$$

$$\Gamma_{\mu\nu} = \begin{cases} \gamma_5\sigma_{\mu\nu} & ;odd \\ \sigma_{\mu\nu} & ;even \end{cases} \tag{187}$$

and the function $\theta_{\mu\nu}(V)$ is:

$$\theta_{\mu\nu}(V) = g_{\mu\nu} - \left(V + \frac{1}{2}\right)\gamma_{\mu}\gamma_{\nu} \ , \tag{188}$$

where $V = X, Y, Z$. Remark that $\theta^{\dagger}_{\mu\nu}(V) = \gamma^{0}\theta_{\nu\mu}(V)\gamma^{0}$.

According to [6], the following relation between the coupling constants holds:

$$\frac{f_{KYR}}{M_K} = \frac{g_{KYR}}{M_R \pm M_Y} \ , \tag{189}$$

with the ± corresponds to the parity of the resonance. N,Y and R are the proton, hyperon and resonance spinors. K stands for the isospin doublet $\begin{pmatrix} K^{+} \\ K^{0} \end{pmatrix}$.

4 The Propagators

Spin 1/2

$$\mathcal{P}_{1/2}(q) = \frac{\not{q} + M}{q^2 - M^2 + iM\Gamma} \ . \tag{190}$$

Spin 3/2

$$\mathcal{P}^{\mu\nu}_{3/2}(q) = \frac{\not{q} + M}{3\left(q^2 - M^2 + iM\Gamma\right)}\left[3g^{\mu\nu} - \gamma^{\mu}\gamma^{\nu} - \frac{2q^{\mu}q^{\nu}}{M^2} - \frac{\gamma^{\mu}q^{\nu} - \gamma^{\nu}q^{\mu}}{M}\right] \ . \tag{191}$$

The hermitian conjugated form of this "Rarita-Schwinger" propagator reads:

$$\left(\mathcal{P}^{\mu\nu}_{3/2}(q)\right)^{\dagger} = \gamma^{0}\left[3g^{\nu\mu} - \gamma^{\nu}\gamma^{\mu} - \frac{2q^{\nu}q^{\mu}}{M^2} + \frac{\gamma^{\nu}q^{\mu} - \gamma^{\mu}q^{\nu}}{M}\right]\frac{\not{q} + M}{3\left(q^2 - M^2 - iM\Gamma\right)}\gamma^{0} \ . \tag{192}$$

Remark the change in sign in the last term between the brackets!

Spin 0

$$\mathcal{P}_0(q) = \frac{1}{q^2 - M^2 + iM\Gamma} \ . \tag{193}$$

Spin 1

$$\mathcal{P}_1^{\mu\nu}(q) = \frac{1}{q^2 - M^2 + iM\Gamma}\left[-g^{\mu\nu} + \frac{q^\mu q^\nu}{M^2}\right] \ . \tag{194}$$

The Eqs.(190), (193) and (194) can be found e.g. in Ref.[1] and many other works. Eq.(191) is given in Ref.[5].

5 The Amplitudes

5.1 Resonance Amplitudes

The amplitude $\mathcal{M}$ is composed by the different amplitudes corresponding to every exchanged particle or resonance in the process. They are build up by the Lagrangians of the vertices and the propagator of the particle. For the case of simplicity, we neglect a factor i in every amplitude. Since,

$$\mathcal{M} = \sum_x i\mathcal{M}_x \qquad\qquad x = s, t, u, K^*, N^*, Y^*; \tag{195}$$

it is clear that:

$$|\mathcal{M}|^2 = \left|\sum_x \mathcal{M}_x\right|^2 \ . \tag{196}$$

5.1.1 Born s-channel

The amplitude for the Born s-channel reads

$$\mathcal{M}_{\mathrm{Born}-s} = e g_{KYp} \overline{U}_Y \gamma_5 \frac{\not{p} + \not{k} + M_p}{s - M_p^2}\left(\gamma^\mu + \frac{\kappa_p}{2M_p} i\sigma^{\mu\nu} k_\nu\right) U_p \varepsilon_\mu \ , \tag{197}$$

and its complex conjugated becomes

$$\mathcal{M}^*_{\mathrm{Born}-s} = - e g_{KYp} \overline{U}_p \left(\gamma^\mu - \frac{\kappa_p}{2M_p} i\sigma^{\mu\nu} k_\nu\right) \frac{\not{p} + \not{k} + M_p}{s - M_p^2} \gamma_5 U_Y \varepsilon_\mu \ . \tag{198}$$

Remark that for all the Born terms, the pseudo-scalar coupling scheme is taken for the $(\Lambda N K)$ vertex.

5.1.2 Born t-channel

For the Born t-channel, the amplitude is

$$\mathcal{M}_{\mathrm{Born}-t} = eg_{KYp}\overline{U}_Y \left(2p_K^\mu - k^\mu\right) \frac{1}{t-M_K^2}\gamma_5 U_p \varepsilon_\mu \ . \tag{199}$$

The complex conjugated amplitude becomes

$$\mathcal{M}^*_{\mathrm{Born}-t} = -\, eg_{KYp}\overline{U}_p\gamma_5 \left(2p_K^\mu - k^\mu\right) \frac{1}{t-M_K^2} U_Y \varepsilon_\mu \ . \tag{200}$$

Remark that a photon can only couple to a charged meson. This implies that the t-channel only exists if $K \equiv K^+$. If neutral kaons are produced and $K \equiv K^0$, the t-channel vanish.

5.1.3 Born u-channel

In the Born u-channel, this is:

$$\mathcal{M}_{\mathrm{Born}-u} = g_{KYp}\overline{U}_Y \left(\gamma^\mu + \frac{e\kappa_Y}{2M_p} i\sigma^{\mu\nu}k_\nu\right) \frac{\not{p}_Y - \not{k} + M_Y}{u-M_Y^2}\gamma_5 U_p \varepsilon_\mu \ , \tag{201}$$

$$\mathcal{M}_{\mathrm{Born}-u-YY'} = g_{KY'p}\overline{U}_Y \frac{e\kappa_{YY'}}{2M_p} i\sigma^{\mu\nu}k_\nu \frac{\not{p}_Y - \not{k} + M_{Y'}}{u-M_{Y'}^2}\gamma_5 U_p \varepsilon_\mu \ , \tag{202}$$

and,

$$\mathcal{M}^*_{\mathrm{Born}-u} = -g_{KYp}\overline{U}_p\gamma_5 \frac{\not{p}_Y - \not{k} + M_Y}{u-M_Y^2} \left(\gamma^\mu - \frac{e\kappa_Y}{2M_p} i\sigma^{\mu\nu}k_\nu\right) U_Y \varepsilon_\mu \ , \tag{203}$$

$$\mathcal{M}^*_{\mathrm{Born}-u-YY'} = g_{KY'p}\overline{U}_p\gamma_5 \frac{\not{p}_Y - \not{k} + M_{Y'}}{u-M_{Y'}^2} \frac{e\kappa_{YY'}}{2M_p} i\sigma^{\mu\nu}k_\nu U_p \varepsilon_\mu \ . \tag{204}$$

Remark that the electric term in Eq. (201) and (203) only appears if charged hyperons are produced ($Y \equiv \Sigma^+$). For neutral particles, there only exists a magnetic coupling.

5.1.4 Vector meson exchange

For vector meson exchange, the amplitude can be cast in a from like:

$$\begin{aligned}\mathcal{M}_{K^*}(P=-) = \; & \frac{eg_{\gamma KK^*}}{M}\overline{U}_Y i\epsilon^{\mu\nu\alpha\beta}k_\nu \left(p_Y - p\right)_\beta \frac{1}{t-M_{K^*}^2 + iM_{K^*}\Gamma_{K^*}} \\ & \times \left(-g_{\alpha\lambda} + \frac{(p_Y-p)_\alpha (p_Y-p)_\lambda}{M_{K^*}^2}\right) \left[g^v_{K^*Yp}\gamma^\lambda + \frac{g^t_{K^*Yp}}{M_Y+M_p} i\sigma^{\lambda\xi}\left(p_Y-p\right)_\xi\right] U_p \varepsilon_\mu \ ,\end{aligned} \tag{205}$$

$$\begin{aligned}\mathcal{M}_{K_1}(P=+) = \; & \frac{eg_{\gamma KK_1}}{M}\overline{U}_Y \left[k_\mu \varepsilon_\alpha \left(p_K - k\right)^\mu - k_\alpha \varepsilon_\nu \left(p_K - k\right)^\nu\right] \frac{1}{t-M_{K_1}^2 + iM_{K_1}\Gamma_{K_1}} \\ & \times \left(-g^{\alpha\lambda} + \frac{(p_Y-p)^\alpha (p_Y-p)^\lambda}{M_{K_1}^2}\right) \left[g^v_{K_1Yp}\gamma_\lambda + \frac{g^t_{K_1Yp}}{M_Y+M_p} i\sigma_{\lambda\xi}\left(p_Y-p\right)^\xi\right] \gamma_5 U_p \ .\end{aligned} \tag{206}$$

Or, simplified and knowing that $(p_Y - p)^\mu = (k - p_K)^\mu$, which cancels the second term in the propagator:

$$\begin{aligned}\mathcal{M}_{K^*}(P=-) &= \overline{U}_Y i\epsilon^{\mu\nu\alpha\beta}k_\nu(-p_K)_\beta \frac{1}{t-M_{K^*}^2+iM_{K^*}\Gamma_{K^*}} \\ &\quad\times(-g_{\alpha\lambda})\left[G_{K^*}^\nu\gamma^\lambda + G_{K^*}^t i\sigma^{\lambda\xi}(p_Y-p)_\xi\right]U_p\varepsilon_\mu \,, \end{aligned} \tag{207}$$

$$\begin{aligned}\mathcal{M}_{K_1}(P=+) &= \overline{U}_Y\left[k_\nu(p_K-k)^\nu\left(-g^{\mu\lambda}\right) - k_\alpha(p_K-k)^\mu\left(-g^{\alpha\lambda}\right)\right] \\ &\quad\times\frac{1}{t-M_{K_1}^2+iM_{K_1}\Gamma_{K_1}}\left[G_{K_1}^\nu\gamma_\lambda + G_{K_1}^t i\sigma_{\lambda\xi}(p_Y-p)^\xi\right]\gamma_5 U_p\varepsilon^\mu \,. \end{aligned} \tag{208}$$

The complex conjugated amplitude becomes:

$$\begin{aligned}\mathcal{M}_{K^*}^*(P=-) &= -\overline{U}_p\left[G_{K^*}^\nu\gamma^\lambda - G_{K^*}^t i\sigma^{\lambda\xi}(p_Y-p)_\xi\right](-g_{\alpha\lambda})\frac{1}{t-M_{K^*}^2-iM_{K^*}\Gamma_{K^*}} \\ &\quad\times\ i\epsilon^{\mu\nu\alpha\beta}k_\nu(-p_K)_\beta U_Y\varepsilon_\mu \,, \end{aligned} \tag{209}$$

$$\begin{aligned}\mathcal{M}_{K_1}^*(P=+) &= -\overline{U}_p\gamma_5\left[G_{K_1}^\nu\gamma_\lambda - G_{K_1}^t i\sigma_{\lambda\xi}(p_Y-p)^\xi\right]\frac{1}{t-M_{K_1}^2-iM_{K_1}\Gamma_{K_1}} \\ &\quad\times\left[k_\nu(p_K-k)^\nu\left(-g^{\mu\lambda}\right) - k_\alpha(p_K-k)^\mu\left(-g^{\alpha\lambda}\right)\right]U_Y\varepsilon^\mu \,. \end{aligned} \tag{210}$$

Herein are the generalized coupling coefficients:

$$\left.\begin{aligned} G_V^\nu &= \tfrac{e g_{\gamma K V}}{M} g_{VYp}^\nu \,, \\ G_V^t &= \tfrac{e g_{\gamma K V}}{M}\tfrac{g_{VYp}^t}{M_Y+M_p} \,, \end{aligned}\right\} V = K^*, K_1; \tag{211}$$

with $M = 1.0$ GeV.

5.1.5 Nucleon spin 1/2 resonance exchange

The nucleon $1/2$ resonance amplitude reads:

$$\begin{aligned}\mathcal{M}_{N^*_{1/2}}(P=+) &= \overline{U}_Y\gamma_5\left[\zeta g_{KYN^*} + (1-\zeta)\frac{f_{KYN^*}}{M_K}p_K^\alpha\gamma_\alpha\right] \\ &\quad\times\frac{\not{p}+\not{k}+M_{N^*}}{s-M_{N^*}^2+iM_{N^*}\Gamma_{N^*}}\frac{e\kappa_{pN^*}}{2M_p}i\sigma_{\mu\nu}k^\nu U_p\varepsilon^\mu \,, \end{aligned} \tag{212}$$

$$\begin{aligned}\mathcal{M}_{N^*_{1/2}}(P=-) &= \overline{U}_Y\left[\zeta g_{KYN^*} - (1-\zeta)\frac{f_{KYN^*}}{M_K}p_K^\alpha\gamma_\alpha\right] \\ &\quad\times\frac{\not{p}+\not{k}+M_{N^*}}{s-M_{N^*}^2+iM_{N^*}\Gamma_{N^*}}\frac{e\kappa_{pN^*}}{2M_p}i\sigma_{\mu\nu}k^\nu\gamma_5 U_p\varepsilon^\mu \,. \end{aligned} \tag{213}$$

Where $P = \pm$ gives the parity of the resonance and ζ is the pseudo-scalar / pseudo-vector mixing parameter. Up to now, we have only worked in the pseudo-scalar ($\zeta = 1$) limit.

After some simplifications, this amplitudes becomes:

$$\mathcal{M}_{N^*_{1/2}}(P=+) = \overline{U}_Y\gamma_5 G_{N^*}\left[\zeta+(1-\zeta)\frac{\not{p}_K}{M_{N^*}+M_Y}\right] \times\frac{\not{p}+\not{k}+M_{N^*}}{s-M_{N^*}^2+iM_{N^*}\Gamma_{N^*}} i\sigma_{\mu\nu}k^\nu U_p\varepsilon^\mu , \tag{214}$$

$$\mathcal{M}_{N^*_{1/2}}(P=-) = \overline{U}_Y G_{N^*}\left[\zeta-(1-\zeta)\frac{\not{p}_K}{M_{N^*}-M_Y}\right] \times\frac{\not{p}+\not{k}+M_{N^*}}{s-M_{N^*}^2+iM_{N^*}\Gamma_{N^*}} i\sigma_{\mu\nu}k^\nu \gamma_5 U_p\varepsilon^\mu . \tag{215}$$

The complex conjugated forms are:

$$\mathcal{M}^*_{N^*_{1/2}}(P=+) = \overline{U}_p G_{N^*} i\sigma_{\mu\nu}k^\nu \frac{\not{p}+\not{k}+M_{N^*}}{s-M_{N^*}^2-iM_{N^*}\Gamma_{N^*}} \times\left[\zeta+(1-\zeta)\frac{\not{p}_K}{M_{N^*}+M_Y}\right]\gamma_5 U_Y\varepsilon^\mu , \tag{216}$$

$$\mathcal{M}^*_{N^*_{1/2}}(P=-) = \overline{U}_p\gamma_5 G_{N^*} i\sigma_{\mu\nu}k^\nu \frac{\not{p}+\not{k}+M_{N^*}}{s-M_{N^*}^2-iM_{N^*}\Gamma_{N^*}} \times\left[\zeta-(1-\zeta)\frac{\not{p}_K}{M_{N^*}-M_Y}\right] U_Y\varepsilon^\mu , \tag{217}$$

with the coupling coefficient:

$$G_{N^*} = g_{KYN^*}\frac{e\kappa_{pN^*}}{2M_p} . \tag{218}$$

5.1.6 Hyperon spin 1/2 resonance exchange

The amplitude for a hyperon 1/2 resonance exchange reads:

$$\mathcal{M}_{Y^*_{1/2}}(P=+) = \overline{U}_Y\frac{e\kappa_{YY^*}}{2M_p} i\sigma_{\mu\nu}k^\nu \frac{\not{p}_Y-\not{k}+M_{Y^*}}{u-M_{Y^*}^2+iM_{Y^*}\Gamma_{Y^*}} \times\left[\zeta g_{KpY^*}-(1-\zeta)\frac{f_{KpY^*}}{M_K}p_K^\eta\gamma_\eta\right]\gamma_5 U_p\varepsilon^\mu , \tag{219}$$

$$\mathcal{M}_{Y^*_{1/2}}(P=-) = \overline{U}_Y\gamma_5\frac{e\kappa_{YY^*}}{2M_p} i\sigma_{\mu\nu}k^\nu \frac{\not{p}_Y-\not{k}+M_{Y^*}}{u-M_{Y^*}^2+iM_{Y^*}\Gamma_{Y^*}} \times\left[\zeta g_{KpY^*}+(1-\zeta)\frac{f_{KpY^*}}{M_K}p_K^\eta\gamma_\eta\right] U_p\varepsilon^\mu . \tag{220}$$

This can be cast in the form:

$$\mathcal{M}_{Y^*_{1/2}}(P=+) = \overline{U}_Y G_{Y^*} i\sigma_{\mu\nu}k^\nu \frac{\not{p}_Y-\not{k}+M_{Y^*}}{u-M_{Y^*}^2+iM_{Y^*}\Gamma_{Y^*}} \times\left[\zeta-(1-\zeta)\frac{\not{p}_K}{M_{Y^*}+M_p}\right]\gamma_5 U_p\varepsilon^\mu , \tag{221}$$

$$\mathcal{M}_{Y^*_{1/2}}(P=-) = \overline{U}_Y \gamma_5 G_{Y^*} i\sigma_{\mu\nu} k^\nu \frac{\not{p}_Y - \not{k} + M_{Y^*}}{u - M^2_{Y^*} + iM_{Y^*}\Gamma_{Y^*}} \times \left[\zeta + (1-\zeta)\frac{\not{p}_K}{M_{Y^*} - M_p}\right] U_p \varepsilon^\mu , \quad (222)$$

and the complex conjugated part as:

$$\mathcal{M}^*_{Y^*_{1/2}}(P=+) = \overline{U}_p \gamma_5 G_{Y^*} \left[\zeta - (1-\zeta)\frac{\not{p}_K}{M_{Y^*} + M_p}\right] \times \frac{\not{p}_Y - \not{k} + M_{Y^*}}{u - M^2_{Y^*} - iM_{Y^*}\Gamma_{Y^*}} i\sigma_{\mu\nu} k^\nu U_Y \varepsilon^\mu , \quad (223)$$

$$\mathcal{M}^*_{Y^*_{1/2}}(P=-) = \overline{U}_p G_{Y^*} \left[\zeta + (1-\zeta)\frac{\not{p}_K}{M_{Y^*} - M_p}\right] \times \frac{\not{p}_Y - \not{k} + M_{Y^*}}{u - M^2_{Y^*} - iM_{Y^*}\Gamma_{Y^*}} i\sigma_{\mu\nu} k^\nu \gamma_5 U_Y \varepsilon^\mu , \quad (224)$$

with the coupling constant

$$G_{Y^*} = g_{KpY^*} \frac{e\kappa_{YY^*}}{2M_p} . \quad (225)$$

5.1.7 Nucleon spin 3/2 resonance exchange

A nucleon 3/2 resonance exchange is described by:

$$\mathcal{M}_{N^*_{3/2}}(P=+) = \frac{f_{KYR}}{M_K} \overline{U}_Y p_K^\alpha \theta_{\alpha\beta}(Z) \mathcal{P}^{\beta\eta}_{3/2}(p+k) \quad (226)$$
$$\times \left[\frac{e\kappa^{(1)}_{pR}}{2M_p}\theta_{\eta\nu}(Y)\gamma_\mu(k^\nu\varepsilon^\mu - k^\mu\varepsilon^\nu) - \frac{e\kappa^{(2)}_{pR}}{4M_p^2}\theta_{\eta\nu}(X)p_\mu(k^\nu\varepsilon^\mu - k^\mu\varepsilon^\nu)\right]\gamma_5 U_p ,$$

$$\mathcal{M}_{N^*_{3/2}}(P=-) = -\frac{f_{KYR}}{M_K} \overline{U}_Y \gamma_5 p_K^\alpha \theta_{\alpha\beta}(Z) \mathcal{P}^{\beta\eta}_{3/2}(p+k) \quad (227)$$
$$\times \left[\frac{e\kappa^{(1)}_{pR}}{2M_p}\theta_{\eta\nu}(Y)\gamma_\mu(k^\nu\varepsilon^\mu - k^\mu\varepsilon^\nu) - \frac{e\kappa^{(2)}_{pR}}{4M_p^2}\theta_{\eta\nu}(X)p_\mu(k^\nu\varepsilon^\mu - k^\mu\varepsilon^\nu)\right] U_p ,$$

or simplified:

$$\mathcal{M}_{N^*_{3/2}}(P=+) = \overline{U}_Y p_K^\alpha \theta_{\alpha\beta}(Z) \mathcal{P}^{\beta\eta}_{3/2}(p+k)\left(G^{(1)}_{N^*}\theta_{\eta\nu}(Y)\gamma_\mu - G^{(2)}_{N^*}\theta_{\eta\nu}(X)p_\mu\right) (k^\nu\varepsilon^\mu - k^\mu\varepsilon^\nu)\gamma_5 U_p ,$$

$$\mathcal{M}_{N^*_{3/2}}(P=-) = -\overline{U}_Y \gamma_5 p_K^\alpha \theta_{\alpha\beta}(Z) \mathcal{P}^{\beta\eta}_{3/2}(p+k)\left(G^{(1)}_{N^*}\theta_{\eta\nu}(Y)\gamma_\mu - G^{(2)}_{N^*}\theta_{\eta\nu}(X)p_\mu\right) (k^\nu\varepsilon^\mu - k^\mu\varepsilon^\nu) U_p ,$$

(228)

The complex conjugated part becomes:

$$\begin{aligned}\mathcal{M}^*_{N^*_{3/2}}(P=+) &= -\overline{U}_p\gamma_5\left(k^\nu\varepsilon^\mu - k^\mu\varepsilon^\nu\right)\left(G^{(1)}_{N^*}\gamma_\mu\theta_{\nu\eta}(Y) - G^{(2)}_{N^*}p_\mu\theta_{\nu\eta}(X)\right)\\ &\quad \left(\mathcal{P}^{\beta\eta}_{3/2}(p+k)\right)^\dagger\theta_{\beta\alpha}(Z)\,p^\alpha_K U_Y\ ,\end{aligned}$$

$$\begin{aligned}\mathcal{M}^*_{N^*_{3/2}}(P=-) &= \overline{U}_p\left(k^\nu\varepsilon^\mu - k^\mu\varepsilon^\nu\right)\left(G^{(1)}_{N^*}\gamma_\mu\theta_{\nu\eta}(Y) - G^{(2)}_{N^*}p_\mu\theta_{\nu\eta}(X)\right)\\ &\quad \left(\mathcal{P}^{\beta\eta}_{3/2}(p+k)\right)^\dagger\theta_{\beta\alpha}(Z)\,p^\alpha_K\gamma_5 U_Y\ ,\end{aligned}$$

where the generalized coupling constants read as:

$$G^{(1)}_{N^*} = \frac{f_{KYN^*}}{M_K}\frac{e\kappa^{(1)}_{pN^*}}{2M_p}\ , \tag{229}$$

$$G^{(2)}_{N^*} = \frac{f_{KYN^*}}{M_K}\frac{e\kappa^{(2)}_{pN^*}}{4M_p^2}\ . \tag{230}$$

5.1.8 Hyperon spin 3/2 resonance exchange

When a hyperon 3/2 resonance is exchanged, the amplitude is:

$$\begin{aligned}\mathcal{M}_{Y^*_{3/2}}(P=+) &= \overline{U}_Y\gamma_5\left[\frac{e\kappa^{(1)}_{YY^*}}{2M_Y}\left(k^\nu\varepsilon^\mu - k^\mu\varepsilon^\nu\right)\gamma_\mu\theta_{\nu\eta}(Y)\right. \\ &\quad \left. - \frac{e\kappa^{(2)}_{YY^*}}{4M_Y^2}\left(k^\nu\varepsilon^\mu - k^\mu\varepsilon^\nu\right)p_{Y\mu}\theta_{\nu\eta}(X)\right]\mathcal{P}^{\eta\alpha}_{3/2}(p_Y-k)\,\frac{f_{KpY^*}}{M_K}\theta_{\alpha\beta}(Z)\,p^\beta_K U_p\ ,\end{aligned} \tag{231}$$

$$\begin{aligned}\mathcal{M}_{Y^*_{3/2}}(P=-) &= -\overline{U}_Y\left[\frac{e\kappa^{(1)}_{YY^*}}{2M_Y}\left(k^\nu\varepsilon^\mu - k^\mu\varepsilon^\nu\right)\gamma_\mu\theta_{\nu\eta}(Y)\right. \\ &\quad \left. - \frac{e\kappa^{(2)}_{YY^*}}{4M_Y^2}\left(k^\nu\varepsilon^\mu - k^\mu\varepsilon^\nu\right)p_{Y\mu}\theta_{\nu\eta}(X)\right]\mathcal{P}^{\eta\alpha}_{3/2}(p_Y-k)\,\frac{f_{KpY^*}}{M_K}\theta_{\alpha\beta}(Z)\,p^\beta_K\gamma_5 U_p\ .\end{aligned} \tag{232}$$

This becomes:

$$\begin{aligned}\mathcal{M}_{Y^*_{3/2}}(P=+) &= \overline{U}_Y\gamma_5\left(k^\nu\varepsilon^\mu - k^\mu\varepsilon^\nu\right)\left(G^{(1)}_{Y^*}\gamma_\mu\theta_{\nu\eta}(Y) - G^{(2)}_{Y^*}p_{Y\mu}\theta_{\nu\eta}(X)\right)\\ &\quad \mathcal{P}^{\eta\alpha}_{3/2}(p_Y-k)\,\theta_{\alpha\beta}(Z)\,p^\beta_K U_p\ ,\end{aligned}$$

$$\begin{aligned}\mathcal{M}_{Y^*_{3/2}}(P=-) &= -\overline{U}_Y\left(k^\nu\varepsilon^\mu - k^\mu\varepsilon^\nu\right)\left(G^{(1)}_{Y^*}\gamma_\mu\theta_{\nu\eta}(Y) - G^{(2)}_{Y^*}p_{Y\mu}\theta_{\nu\eta}(X)\right)\\ &\quad \mathcal{P}^{\eta\alpha}_{3/2}(p_Y-k)\,\theta_{\alpha\beta}(Z)\,p^\beta_K\gamma_5 U_p\ .\end{aligned}$$

Its complex conjugated is

$$\mathcal{M}^*_{Y^*_{3/2}}(P=+) = -\overline{U}_p p^\beta_K\theta_{\beta\alpha}(Z)\left(\mathcal{P}^{\eta\alpha}_{3/2}(p_Y-k)\right)^\dagger$$

$$\left(G^{(1)}_{Y^*}\theta_{\eta\nu}(Y)\gamma_\mu - G^{(2)}_{Y^*}\theta_{\eta\nu}(X)p_{Y\mu}\right)(k^\nu\varepsilon^\mu - k^\mu\varepsilon^\nu)\gamma_5 U_Y\ , \tag{233}$$

$$\begin{aligned}\mathcal{M}^*_{Y^*_{3/2}}(P=-) &= \overline{U}_p\gamma_5 p^\beta_K\theta_{\beta\alpha}(Z)\left(\mathcal{P}^{\eta\alpha}_{3/2}(p_Y-k)\right)^\dagger \\ &\quad \left(G^{(1)}_{Y^*}\theta_{\eta\nu}(Y)\gamma_\mu - G^{(2)}_{Y^*}\theta_{\eta\nu}(X)p_{Y\mu}\right)(k^\nu\varepsilon^\mu - k^\mu\varepsilon^\nu)U_Y\ .\end{aligned} \tag{234}$$

The generalized coupling coefficients read:

$$G^{(1)}_{Y^*} = \frac{f_{KpY^*}}{M_K}\frac{e\kappa^{(1)}_{YY^*}}{2M_Y}\ , \tag{235}$$

$$G^{(2)}_{Y^*} = \frac{f_{KpY^*}}{M_K}\frac{e\kappa^{(2)}_{YY^*}}{4M_Y^2}\ . \tag{236}$$

5.2 Normalization of the coupling constants

Each of the interaction Lagrangians given in Section 3, depends on a coupling constants, sometimes normalized on a specified mass. In Table 3 we summarize which raw numbers enters the code through the file `coupl.iso.*` and how they are normalized during the calculations. This normalization is important if values for coupling constants are compared to other model calculations. Remark that there is some ambiguity for the spin 3/2 particles. At present, there are two normalization schemes available.

5.3 The Lorentz and Gauge invariant Representation

In the previous subsection, we build up the amplitude $|\mathcal{M}|^2$ at tree level starting from the Lagrangians and the Feynman diagrams. This amplitudes for meson **photo**production processes can also be cast in a complete Lorentz and gauge invariant way. We can write:

$$\mathcal{M} = i\,\overline{U}_Y\sum_{j=1}^{4}\mathcal{A}_j\mathcal{M}_j U_p\ . \tag{237}$$

The Lorentz and gauge invariant matrices are given by:

$$\mathcal{M}_1 = -\gamma_5\ \not\varepsilon\ \not k\ , \tag{238}$$

$$\mathcal{M}_2 = 2\gamma_5\left[(\varepsilon\cdot p)(k\cdot p_Y) - (\varepsilon\cdot p_Y)(k\cdot p)\right]\ , \tag{239}$$

$$\mathcal{M}_3 = \gamma_5\left[\not\varepsilon(k\cdot p) - \not k(\varepsilon\cdot p)\right]\ , \tag{240}$$

$$\mathcal{M}_4 = \gamma_5\left[\not\varepsilon(k\cdot p_Y) - \not k(\varepsilon\cdot p_Y)\right]\ . \tag{241}$$

Remark that for the **electro**production process two additional invariant matrices have to be included.

The contributions of the different resonance amplitudes to $\mathcal{A}_j$ are given in the following sections.

Diagram	Raw coupling constant	Lagrangian coupling constant new	old
Born terms	$\frac{g_{KYp}}{\sqrt{4\pi}}$	$\frac{eg_{KYp}}{4\pi}$	-
	κ_B	$\frac{eg_{KYp}}{4\pi}\frac{\kappa_B}{2M_p}$	-
Vector meson	$\frac{eg^{v}_{VYp}}{4\pi}\kappa_{KV}$	$\frac{eg^{v}_{VYp}}{4\pi}\frac{\kappa_{KV}}{M}$	-
	$\frac{eg^{t}_{VYp}}{4\pi}\kappa_{KV}$	$\frac{eg^{t}_{VYp}}{4\pi(M_p+M_Y)}\frac{\kappa_{KV}}{M}$	-
Spin 1/2 resonance	$\frac{g_{KBR}}{\sqrt{4\pi}}\kappa_{BR}$	$\frac{eg_{KBR}}{4\pi}\frac{\kappa_{BR}}{2M_p}$	-
Spin 3/2 resonance	$\frac{ef_{KBR}}{4\pi}\kappa^{(1)}_{BR}$	$\frac{ef_{KBR}}{4\pi M_K}\frac{\kappa^{(1)}_{BR}}{2M_B}$	$\frac{ef_{KBR}}{4\pi M_R}\frac{\kappa^{(1)}_{BR}}{(M_B+M_R)}$
	$\frac{ef_{KBR}}{4\pi}\kappa^{(2)}_{BR}$	$\frac{ef_{KBR}}{4\pi M_K}\frac{\kappa^{(2)}_{BR}}{4M_B^2}$	$\frac{ef_{KBR}}{4\pi M_R}\frac{\kappa^{(2)}_{BR}}{(M_B+M_R)^2}$

Table 3: Normalization of the coupling constants. The raw numbers enters the code, the normalized ones are those given by the Lagrangians. For the spin 3/2 particles, the `new` and the `old` normalization schemes are given

5.3.1 Born Terms

$$\mathcal{A}_1 = \frac{eg_{KYp}}{s - M_p^2}\left(1+\kappa_p\right) + \frac{eg_{KYp}}{u - M_Y^2}\frac{M_Y}{M_p}\kappa_Y \ , \tag{242}$$

$$\mathcal{A}_2 = \frac{2eg_{KYp}}{\left(s - M_p^2\right)\left(t - M_K^2\right)} \ , \tag{243}$$

$$\mathcal{A}_3 = \frac{eg_{KYp}}{s - M_p^2}\frac{1}{M_p}\kappa_p \ , \tag{244}$$

$$\mathcal{A}_4 = \frac{eg_{KYp}}{u - M_Y^2}\frac{1}{M_p}\kappa_Y \ . \tag{245}$$

5.3.2 Born YY′ $\left(\equiv \left(\Lambda, \Sigma^0\right), \left(\Sigma^0, \Lambda\right)\right)$ Term

$$\mathcal{A}_1 = \frac{eg_{KY'p}}{u - M_{Y'}^2}\frac{M_{Y'} + M_Y}{2M_p}\kappa_{YY'} \ , \tag{246}$$

$$\mathcal{A}_2 = 0 \ , \tag{247}$$

$$\mathcal{A}_3 = 0 \ , \tag{248}$$

$$\mathcal{A}_4 = \frac{eg_{KY'p}}{u - M_{Y'}^2}\frac{1}{M_p}\kappa_{YY'} \ . \tag{249}$$

5.3.3 K^* Vector Meson Exchange

$$\mathcal{A}_1 = \frac{1}{t - M_{K^*}^2 + iM_{K^*}\Gamma_{K^*}}\left[G_{K^*}^{v}\left(M_Y + M_p\right) + G_{K^*}^{t}t\right] \ , \tag{250}$$

$$\mathcal{A}_2 = \frac{1}{t - M_{K^*}^2 + iM_{K^*}\Gamma_{K^*}}G_{K^*}^{t} \ , \tag{251}$$

$$\mathcal{A}_3 = \frac{1}{t - M_{K^*}^2 + iM_{K^*}\Gamma_{K^*}}\left[G_{K^*}^{v} - G_{K^*}^{t}\left(M_Y - M_p\right)\right] \ , \tag{252}$$

$$\mathcal{A}_4 = \frac{1}{t - M_{K^*}^2 + iM_{K^*}\Gamma_{K^*}}\left[G_{K^*}^{v} + G_{K^*}^{t}\left(M_Y - M_p\right)\right] \ . \tag{253}$$

5.3.4 K_1 Vector Meson Exchange

$$\mathcal{A}_1 = 0 \ , \tag{254}$$

$$\mathcal{A}_2 = -\frac{1}{t - M_{K_1}^2 + iM_{K_1}\Gamma_{K_1}}G_{K_1}^{t} \ , \tag{255}$$

$$\mathcal{A}_3 = \frac{1}{t - M_{K_1}^2 + iM_{K_1}\Gamma_{K_1}}\left[G_{K_1}^{v} + G_{K_1}^{t}\left(M_Y - M_p\right)\right] \ , \tag{256}$$

$$\mathcal{A}_4 = -\frac{1}{t - M_{K_1}^2 + iM_{K_1}\Gamma_{K_1}}\left[G_{K_1}^{v} + G_{K_1}^{t}\left(M_Y - M_p\right)\right] \ . \tag{257}$$

5.3.5 $N^*_{1/2}\,(\pm)$ Exchange

$$
\begin{aligned}
\mathcal{A}_1 &= \frac{e g_{KYN^*}}{s - M_{N^*}^2 + iM_{N^*}\Gamma_{N^*}} \frac{M_{N^*} \pm M_p}{2M_p} \kappa_{pN^*} \,, & (258)\\
\mathcal{A}_2 &= 0 \,, & (259)\\
\mathcal{A}_3 &= \pm \frac{e g_{KYN^*}}{s - M_{N^*}^2 + iM_{N^*}\Gamma_{N^*}} \frac{1}{M_p} \kappa_{pN^*} \,, & (260)\\
\mathcal{A}_4 &= 0 \,. & (261)
\end{aligned}
$$

5.3.6 $Y^*_{1/2}\,(\pm)$ Exchange

$$
\begin{aligned}
\mathcal{A}_1 &= \frac{e g_{KpY^*}}{u - M_{Y^*}^2 + iM_{Y^*}\Gamma_{Y^*}} \frac{M_{Y^*} \pm M_Y}{2M_p} \kappa_{YY^*} \,, & (262)\\
\mathcal{A}_2 &= 0 \,, & (263)\\
\mathcal{A}_3 &= 0 \,, & (264)\\
\mathcal{A}_4 &= \pm \frac{e g_{KpY^*}}{u - M_{Y^*}^2 + iM_{Y^*}\Gamma_{Y^*}} \frac{1}{M_p} \kappa_{YY^*} \,. & (265)
\end{aligned}
$$

6 Decay/Helicity amplitudes and coupling constants

In our Lagrangian formalism, the coupling constants of the intermediate particles are the relevant quantities. Nevertheless, there are other objects, related to our coupling constants, which are frequently used to compare different model calculations. For the electromagnetic decay process of the resonances, this are the photocoupling helicity amplitudes. For the hadronic process, this is the hadronic decay amplitude. For the latter, one also frequently compares hadronic decay widths. We will come back to this point in Sec. 7.

6.1 Electromagnetic coupling

6.1.1 Spin 1/2 resonance exchange

The electromagnetic decay of a spin 1/2 resonance is described by the interaction Lagrangian of Eq. (168) which essentially depends on the magnetic transition moment κ_{BR}. This coupling can also be expressed by the so called photocoupling helicity amplitude which is given by:

$$
A_{1/2} = \mp \frac{e}{2M_N} \sqrt{\frac{M_R^2 - M_N^2}{2M_N}} \kappa_{NR} \,. \qquad (266)
$$

The experimental values for $A_{1/2}$ are given in Table 4.

Notation	Resonance	$A^n_{1/2}$	$A^p_{1/2}$	$A^n_{3/2}$	$A^p_{3/2}$
N1	N(1440)	40 (±10)	−65 (±4)	–	–
N2	N(1520)	−59 (±9)	−24 (±9)	−139 (±11)	166 (±5)
N3	N(1535)	−46 (±27)	90 (±30)	–	–
N4	N(1650)	−15 (±21)	53 (±16)	–	–
N5	N(1700)	0 (±50)	−18 (±13)	−3 (±44)	−2 (±24)
N6	N(1710)	−2 (±14)	9 (±22)	–	–
N7	N(1720)	1 (±15)	18 (±30)	−29 (±61)	−19 (±20)
N8	N(1895)	?	?	?	?

Table 4: Photocoupling helicity amplitudes for the nucleon. The numeric values are in $\mathrm{GeV}^{-1/2}$ and taken from Ref.[34]

6.1.2 Spin 3/2 resonance exchange

The electromagnetic decay of a spin 1/2 resonance is described by the interaction Lagrangians of Eq. (171) and (172). The coupling is described by two magnetic transition moment $\kappa^{(1)}_{BR}$ and $\kappa^{(2)}_{BR}$. Since the coupling of a spin 1 photon and a spin 1/2 nucleon to a spin 3/2 resonance can be constructed in two different ways, there are two helicity amplitudes:

$$A_{1/2} = \frac{e}{4M_R}\sqrt{\frac{M_R^2 - M_N^2}{3M_N}}\left(\pm\,\kappa^{(1)}_{NR} + \frac{M_R}{4M_N^2}\,(M_R \mp M_N)\,\kappa^{(2)}_{NR}\right), \quad (267)$$

$$A_{3/2} = \frac{e}{4M_N}\sqrt{\frac{M_R^2 - M_N^2}{M_N}}\left(\pm\,\kappa^{(1)}_{NR} \mp \frac{1}{4M_N}\,(M_R \mp M_N)\,\kappa^{(2)}_{NR}\right). \quad (268)$$

Note there is an additional sign needed in both formula compared tot Feuster and Mosel [16] since they used an extra sign in their Lagrangian for the photo coupling to a spin 3/2 resonance. These relations can be inverted to obtain the transition moments from the measured helicity amplitudes:

$$e\kappa^{(1)}_{NR} = \frac{4M_N M_R}{M_R \pm M_N}\sqrt{\frac{M_N}{M_R^2 - M_N^2}}\left(\sqrt{3}A_{1/2} \pm A_{3/2}\right), \quad (269)$$

$$e\kappa^{(2)}_{NR} = \frac{16M_N^2 M_R}{M_R^2 - M_N^2}\sqrt{\frac{M_N}{M_R^2 - M_N^2}}\left(\sqrt{3}A_{1/2} - \frac{M_N}{M_R}A_{3/2}\right). \quad (270)$$

6.2 Hadronic decay

For hadronic decay, it is common to define the decay amplitude as [24, 10]:

$$\mathcal{T} = \zeta_{in}\zeta_{out}\sqrt{\Gamma_{R\to KY}} \, . \tag{271}$$

Herein is $\zeta_{in(out)}$ the sign of the incoming (outgoing) amplitude. Here, the product $\zeta_{in}\zeta_{out}$ is given by the sign of the hadronic coupling constant g_{RKY}. This expression is valid for spin 1/2 and spin 3/2 resonance decay. The decay widths, and their connection to the Lagrangians will be discussed in Sec. 7.

7 Decay widths and coupling constants

From the interaction Lagrangians summarized in Section 3, not only the reaction amplitudes can be calculated, but also the decay width of a resonance in a specific final state can be determined. To obtain these decay widths, we start from the general expression [35]:

$$\Gamma = \frac{1}{2M_R}\int \frac{d^3\vec{p}_1}{(2\pi)^3}\frac{1}{2E_1}\frac{d^3\vec{p}_2}{(2\pi)^3}\frac{1}{2E_2}\,(2\pi)^4\,\delta^{(4)}\left(p_R - p_1 - p_2\right)\left|\mathcal{M}\left(R\to 1,2\right)\right|^2 \, , \tag{272}$$

where 1 and 2 are the decay products of the resonance R. Carrying out the integration over the phase space, this formula reduces into:

$$\Gamma = \frac{|\vec{p}_1|}{8\pi M_R^2}\left|\mathcal{M}\left(R\to 1,2\right)\right|^2 \, . \tag{273}$$

Note that this expression holds for hadronic as well as electromagnetic decay in the center of mass frame of the two escaping particles (= $\vec{p}_1 = -\vec{p}_2$), with the resonance put on-shell.

7.1 The kinematics

Solving the kinematics for the decay process in the center of mass frame, we end up with the subsequent equations for the energies for the two escaping particles:

$$E_1 = \frac{M_R^2 + M_1^2 - M_2^2}{2M_R} \, , \tag{274}$$

$$E_2 = \frac{M_R^2 + M_2^2 - M_1^2}{2M_R} \, . \tag{275}$$

For the center of mass momentum $|\vec{p}| = |\vec{p}_1| = |\vec{p}_2|$ one obtains:

$$|\vec{p}| = \frac{\sqrt{M_R^4 + M_1^4 + M_2^4 - 2\left(M_R^2 M_1^2 + M_R^2 M_2^2 + M_1^2 M_2^2\right)}}{2M_R} \, . \tag{276}$$

In the next sections, these expression need to be evaluated for the case of baryon-meson, baryon-photon or meson-photon decay.

7.2 Spin 1/2 resonance decay

7.2.1 Hadronic decay

Remark that for the hadronic vertex of a spin 1/2 particle, two coupling schemes are available. As far as the resonance is treated on-shell, both schemes give the same result. From Eq. (169) one can construct the decay amplitude:

$$\mathcal{M} = -ig_{RKY}\overline{u}_Y \Gamma u_R \ . \tag{277}$$

In combination with Eq. (273) the decay width becomes:

$$\Gamma_{R\to KY} = \frac{g_{RKY}^2}{4\pi}\,|\vec{p}_K|\,\frac{E_Y \mp M_Y}{M_R} \ . \tag{278}$$

To obtain the decay width from the amplitudes $\mathcal{T}$ given by the work of Capstick *et al.* [10] or Kroniuk [24], we obtain from Eq. (271):

$$\Gamma_{R\to KY} = \mathcal{T}^2 \ . \tag{279}$$

7.2.2 Electromagnetic decay

The electromagnetic decay amplitude can be constructed from Eq. (168):

$$\mathcal{M} = \pm i\,\frac{e\kappa_{pR}}{2M_p}\,\overline{u}_p \Gamma_{\mu\nu} k^\nu u_R \varepsilon^\mu \ , \tag{280}$$

and the decay width becomes:

$$\Gamma_{R\to p\gamma} = \left(\frac{e\kappa_{pR}}{2M_p}\right)^2 \frac{\omega^2}{2\pi}\left(\frac{M_R^2 - M_p^2}{M_R}\right) \ . \tag{281}$$

This expression is compatible with the relation:

$$\Gamma_{R\to p\gamma} = \frac{\omega^2}{\pi}\frac{M_p}{M_R}\left|A_{1/2}\right|^2 \ , \tag{282}$$

where the photo helicity coupling is given in Eq. (266).

7.2.3 Width versus Coupling constant

The fractional decay width is connected to the coupling constants by:

$$\Gamma_{R\to KY}\Gamma_{R\to p\gamma} = G_R^2\left(\frac{e^2}{4\pi}\right)|\vec{p}_K|\,\omega^2\left(\frac{E_Y \mp M_Y}{2}\right)\left(\frac{M_R^2 - M_p^2}{M_R^2 M_p^2}\right) \ , \tag{283}$$

where G_R, the code input coupling constant, is given by:

$$G_R = \frac{g_{RKY}\kappa_{pR}}{\sqrt{4\pi}} \ . \tag{284}$$

7.3 Spin 3/2 resonance decay

7.3.1 Hadronic decay

According to Eq. (173) the hadronic decay amplitude reads:

$$\mathcal{M} = \pm i \, \frac{f_{KYR}}{M_K} \, \bar{u}_Y p_K^\nu \Gamma' \theta_{\nu\mu} (Z) \, u_R^\mu \, . \tag{285}$$

Note that since we treat the resonances on-shell, the off-shell contribution have no effect on the final result and $\theta_{\nu\mu}(Z)$ reduces to $g_{\nu\mu}$. The decay width is given by:

$$\Gamma_{R\to KY} = \left(\frac{f_{KYR}}{M_K}\right)^2 \frac{|\vec{p}_K|^3}{12\pi} \left(\frac{E_Y \pm M_Y}{M_R}\right) \, . \tag{286}$$

ATTENTION: Remark that in the calculation of this decay width, there is some confusion about the sign in the subsequent equation:

$$\sum_{\text{spins}} u_R^\mu \bar{u}_R^\nu = -P_{3/2}^{\mu\nu}(p_R) \ , \tag{287}$$

where $P_{3/2}^{\mu\nu}(p_R)$ is given by the nominator of the Rarita-Schwinger propagator in Eq. (191). If a positive sign is taken, the one that one would expect from the propagator form, a negative decay width is obtained. So, in the following calculations, the minus sign is used. To derive the width from the decay amplitude given by Capstick or Kroniuk, we obtain from Eq. (271):

$$\Gamma_{R\to KY} = \mathcal{T}^2 \, . \tag{288}$$

7.3.2 Electromagnetic decay

For the electromagnetic decay of a spin 3/2 resonance, the Lagrangian falls apart in two parts. The corresponding decay amplitudes reads:

$$\begin{aligned} \mathcal{M} &= \frac{e\kappa_{pR}^{(1)}}{2M_p} \, \bar{u}_p \left(\Gamma_\lambda k_\mu - \Gamma_\nu k^\nu g_{\mu\lambda}\right) u_R^\mu \varepsilon^\lambda \\ &\pm \frac{e\kappa_{pR}^{(2)}}{4M_p^2} \, \bar{u}_p \Gamma \left(p_\lambda g_{\nu\mu} - p_\nu g_{\lambda\mu}\right) k^\nu u_R^\mu \varepsilon^\lambda \, . \end{aligned} \tag{289}$$

The corresponding decay widths are given by:

$$\Gamma_{R\to p\gamma} = \frac{\omega^3}{6\pi} \left[C_1^2 \left(\frac{E_p + M_R}{M_R}\right) + C_2^2 M_R \left(E_p \mp M_p\right) - C_1 C_2 \left(\omega + 2\left(E_p \mp M_p\right)\right) \right] \ , \tag{290}$$

where the generalized coupling constants are given by:

$$C_1 = \frac{e\kappa_{pR}^{(1)}}{2M_p} \ , \tag{291}$$

$$C_2 = \frac{e\kappa_{pR}^{(2)}}{4M_p^2} \, . \tag{292}$$

Note that this expression for the decay width is compatible with the formula:

$$\Gamma_{R\rightarrow p\gamma} = \frac{\omega^2}{\pi}\frac{M_p}{2M_R}\left(\left|A_{1/2}\right|^2 + \left|A_{3/2}\right|^2\right) , \tag{293}$$

Where the photo helicity amplitudes are given in Eq. (267) and (268).

7.3.3 Width versus Coupling constant

The fractional decay width is connected to the coupling constants by:

$$\begin{aligned}\Gamma_{R\rightarrow KY}\Gamma_{R\rightarrow p\gamma} &= \frac{|\vec{p}_K|^3\,\omega^3}{18M_p^2M_K^2M_R^2}\left(E_Y \pm M_Y\right)\Big[G_{(1)}^2\left(E_p + M_R\right) \\ &+G_{(2)}^2\frac{M_R^2}{4M_p^2}\left(E_p \mp M_p\right) - G_{(1)}G_{(2)}\frac{M_R}{2M_p}\left(\omega + 2\left(E_p \mp M_p\right)\right)\Big] ,\end{aligned} \tag{294}$$

where the code input coupling constants are given by:

$$G_{(1)} = \frac{e\kappa_{pR}^{(1)}f_{RKY}}{4\pi} , \tag{295}$$

$$G_{(2)} = \frac{e\kappa_{pR}^{(2)}f_{RKY}}{4\pi} . \tag{296}$$

7.4 Vector meson decay

7.4.1 K^* radiative decay

The transition amplitude for electromagnetic K^* decay reads:

$$\mathcal{M} = -i\,\frac{eg_{\gamma KK^*}}{4M} i\epsilon^{\mu\nu\alpha\beta}k_\nu\varepsilon_\mu p_\beta^{K^*}\phi_\alpha^{K^*} , \tag{297}$$

and the decay width is given by:

$$\Gamma_{K^*\rightarrow K\gamma} = \left(\frac{eg_{\gamma KK^*}}{M}\right)^2\frac{|\vec{p}_K|^3}{12\pi} . \tag{298}$$

8 Form Factors & Gauge Invariance

Introducing form factors to describe the internal structure of the vertices, causes problems to ensure gauge invariance. Since the amplitudes of the resonances are gauge invariant by them self, introduction of a four momentum dependent coupling constant or form factor does not affect current conservation. Problems arise for the born terms, since here only the total sum of the amplitudes of the s-, t- and u-channel is gauge invariant and introduction of different form factors in different channels breaks current conservation.

Form factors can either be introduced at the electromagnetic or at the hadronic vertices to incorporate internal structure. Both types need a different treatment.

8.1 Electromagnetic Vertices

8.1.1 Gauge Invariance

Concerns about gauge invariance and electromagnetic form factors are only necessary in the electroproduction case. In the real photon limit, the Q^2 dependent form factors reduce to unit or zero and gauge invariance is ensured.
The electromagnetic vertex functions for the electroproduction process take the form:

$$\mathcal{V}(\gamma pp) = e\left(F_1^p\left(Q^2\right)\gamma_\mu + F_2^p\left(Q^2\right)\frac{\kappa_p}{2M_p}i\sigma_{\mu\nu}k^\nu\right), \tag{299}$$

$$\mathcal{V}(\gamma YY) = e\left(F_1^Y\left(Q^2\right)\gamma_\mu + F_2^Y\left(Q^2\right)\frac{\kappa_Y}{2M_p}i\sigma_{\mu\nu}k^\nu\right), \tag{300}$$

$$\mathcal{V}(\gamma KK) = e\,F_K\left(Q^2\right)(2p_K - k)_\mu\ . \tag{301}$$

The two "spin" or Pauli currents with the F_2 form factor preserve gauge invariance by them self since these currents are purely transverse. The Dirac coupling $(e\gamma_\mu)$ breaks gauge invariance and can not be restored with the aid of the t-channel in the case of K^+Y (with $Y = \Lambda^0, \Sigma^0$) or the u-channel in the case of $K^0\Sigma^+$ processes, as is possible in the real photon limit.
To restore current conservation, we follow the procedure of Nozawa and Lee [33] based on the Ward-Takahashi identity. The suggested replacement is:

$$F_1^{p,Y}\left(Q^2\right)\gamma_\mu \longrightarrow F_1^{p,Y}\left(Q^2\right)\left(\gamma_\mu + \frac{\not{k}}{Q^2}k_\mu\right) - F_1^{p,Y}(0)\,\frac{\not{k}}{Q^2}k_\mu\ , \tag{302}$$

$$F_K\left(Q^2\right)(2p_K - k)_\mu \longrightarrow F_K\left(Q^2\right)\left((2p_K - k)_\mu + \frac{(2p_K - k)\cdot k}{Q^2}k_\mu\right) - F_K(0)\left(\frac{(2p_K - k)\cdot k}{Q^2}k_\mu\right)\ . \tag{303}$$

Nozawa and Lee showed [33] that this procedure can be related to the vector meson dominance model. While dotting these expressions with k^μ to proof current conservation, it becomes clear that one falls back on the proof for the real photon limit, which is easily obtained with the aid of the Dirac equation.

It is important to remark that the additional terms, introduced to ensure gauge invariance, do not contribute to the cross section. All these terms are orthogonal to the lepton current. This becomes clear when one remind (see Eq.(79)) that the hadronic tensor is composed by the contraction of the hadronic current en the virtual photon polarization four vector ε_μ. Since all the gauge restoring terms are proportional to k^μ and for the massive photon holds $(k \cdot \varepsilon) = 0$, these terms vanish identically.

8.1.2 Baryonic Form Factors

The (γpp) vertex (see Eq.(302)) is extensively studied and in the literature, different parameterizations are used. We have introduced the Dirac (F_1^p) and the Pauli (F_2^p) form factors which are related to the Sachs electric and magnetic form factors by:

$$F_1^p = \frac{1}{1+\tau}\left(G_E^p + \tau G_M^p\right) , \quad (304)$$

$$F_2^p = \frac{1}{\kappa_p (1+\tau)}\left(G_M^p - G_E^p\right) , \quad (305)$$

where $\tau = Q^2/4M_p^2$. We have retained the model developed by Gari and Krümpelmann [18, 19, 20] that is based on the extended vector meson dominance model. This model combines the low Q^2 vector meson dominance hypothesis with the high Q^2 perturbative QCD approach. According to [19] and reexamined by [27], the form factors are expressed in terms of isoscalar (*is*) and isovector (*iv*) parts:

$$F_1^p = \frac{1}{2}\left(F_1^{is} + F_1^{iv}\right) , \quad (306)$$

$$F_2^p = \frac{1}{2\kappa_p}\left(\kappa_{is} F_2^{is} + \kappa_{iv} F_2^{iv}\right) , \quad (307)$$

where this parts take the form:

$$F_1^{iv}\left(Q^2\right) = \frac{g_\rho}{f_\rho}\frac{m_\rho^2}{m_\rho^2 + Q^2} F_1^\rho\left(Q^2\right) + \left(1 - \frac{g_\rho}{f_\rho}\right) F_1^D\left(Q^2\right) , \quad (308)$$

$$\kappa_{iv} F_2^{iv}\left(Q^2\right) = \kappa_\rho \frac{g_\rho}{f_\rho}\frac{m_\rho^2}{m_\rho^2 + Q^2} F_2^\rho\left(Q^2\right) + \left(\kappa_{iv} - \kappa_\rho \frac{g_\rho}{f_\rho}\right) F_2^D\left(Q^2\right) , \quad (309)$$

$$F_1^{is}\left(Q^2\right) = \frac{g_\omega}{f_\omega}\frac{m_\omega^2}{m_\omega^2 + Q^2} F_1^\omega\left(Q^2\right) + \frac{g_\phi}{f_\phi}\frac{m_\phi^2}{m_\phi^2 + Q^2} F_1^\phi\left(Q^2\right) + \left(1 - \frac{g_\omega}{f_\omega}\right) F_1^D\left(Q^2\right) , \quad (310)$$

$$\kappa_{is} F_2^{is}\left(Q^2\right) = \kappa_\omega \frac{g_\omega}{f_\omega}\frac{m_\omega^2}{m_\omega^2 + Q^2} F_2^\omega\left(Q^2\right) + \kappa_\phi \frac{g_\phi}{f_\phi}\frac{m_\phi^2}{m_\phi^2 + Q^2} F_2^\phi\left(Q^2\right) + \left(\kappa_{is} - \kappa_\omega \frac{g_\omega}{f_\omega} - \kappa_\phi \frac{g_\phi}{f_\phi}\right) F_2^D\left(Q^2\right) . \quad (311)$$

Here, g_ρ, g_ω and g_ϕ are the vector meson-nucleon coupling constants, m_ρ^2/f_ρ, m_ω^2/f_ω and m_ϕ^2/f_ϕ gives the coupling of a photon to the vector mesons and the κ's are the magnetic moments. F_i^ρ, F_i^ω and F_i^ϕ denote the meson-nucleon form factors and F_i^D describes the nucleon non resonant quark structure, which is responsible for the asymptotic behavior at $Q^2 \to \infty$. Remark that in [19] the ϕ-term is not included!

In order to have a smooth transition between the low and high Q^2 domains, the following simple form is used:

$$F_1^\alpha\left(Q^2\right) = \frac{\Lambda_1^2}{\Lambda_1^2 + \widetilde{Q}^2}\frac{\Lambda_2^2}{\Lambda_2^2 + \widetilde{Q}^2} , \quad (312)$$

κ_{iv}	κ_{is}	g_ρ/f_ρ	κ_ρ	g_ω/f_ω	κ_ω	g_ϕ/f_ϕ	κ_ϕ	μ_ϕ	$\Lambda_1^{\rho,\omega}$	Λ_1^D	Λ_2	Λ_{QCD}
3.706	-0.12	0.631	3.3	0.658	0.4	-	-	-	0.863	1.21	2.1	0.33

Table 5: Parameters for the nucleonic electromagnetic form factors obtained by Gari and Krümpelmann [19]. The Λ's are given in GeV. Remark that the ϕ-term is not included.

$$F_2^\alpha\left(Q^2\right) = \left[\frac{\Lambda_1^2}{\Lambda_1^2+\widetilde{Q}^2}\right]^2 \frac{\Lambda_2^2}{\Lambda_2^2+\widetilde{Q}^2}\,, \tag{313}$$

$$F_1^\phi\left(Q^2\right) = F_1^\alpha\left(Q^2\right)\left[\frac{Q^2}{\Lambda_1^2+Q^2}\right]^{1.5}\,, \tag{314}$$

$$F_2^\phi\left(Q^2\right) = F_2^\alpha\left(Q^2\right)\left[\frac{\Lambda_1^2}{\mu_\phi^2}\frac{\mu_\phi^2+Q^2}{\Lambda_1^2+Q^2}\right]^{1.5}\,, \tag{315}$$

with $\alpha=\rho,\omega,D$ and

$$\widetilde{Q}^2 = Q^2\ln\left(\frac{\Lambda_2^2+Q^2}{\Lambda_{QCD}^2}\right)\Big/\ln\left(\frac{\Lambda_2^2}{\Lambda_{QCD}^2}\right)\,. \tag{316}$$

The values of the parameters obtained by Gari and Krümpelmann [19] are summarized in Table 5. The reexamined values of Lomon [27] are given in Table 6. Remark that in this latter paper, more complicated forms are proposed.

For the $(\gamma Y\Lambda)$ and $(\gamma Y\Sigma^0)$ vertices $(Y=\Lambda,\Sigma^0)$, we have used the two form factors of the neutron:

$$F_1^Y = F_1^n = \frac{1}{2}\left(F_1^{is}-F_1^{iv}\right)\,, \tag{317}$$

$$F_2^Y = F_2^n = \frac{1}{2\kappa_n}\left(\kappa_{is}F_2^{is}-\kappa_{iv}F_2^{iv}\right)\,, \tag{318}$$

where the isoscalar and isovector parts are defined in Eq.(308-311).

For the nucleonic (hyperonic) resonances, we take the Pauli form factor of the proton (neutron). Namely, F_2^p at (γpN^*) vertices and F_2^n at $(\gamma\Lambda Y^*)$ and $(\gamma\Sigma^0 Y^*)$.

As an other option, we can take a dipole form for those nucleonic and hyperonic resonances. This kind of form reads:

$$\left(\frac{1}{1+Q^2/\Lambda^2}\right)^2\,. \tag{319}$$

κ_{iv}	κ_{is}	g_ρ/f_ρ	κ_ρ	g_ω/f_ω	κ_ω	g_ϕ/f_ϕ	κ_ϕ	μ_ϕ
3.706	-0.12	0.4466	4.3472	0.4713	21.762	-0.8461	11.849	1.1498
		$\Lambda_1^{\rho,\omega}$	Λ_1^D	Λ_2	Λ_{QCD}			
		0.9006	1.7038	1.1336	0.0312			

Table 6: Parameters for the nucleonic electromagnetic form factors obtained by Lomon [27]. The Λ's are given in GeV.

With Λ = 840 MeV, this form come close to the parametrization of Gari and Krümpelmann.

8.1.3 Kaonic Form Factors

For the $(\gamma K^+ K^+)$ vertex we use a form factor derived within a relativistic constituent quark model based on the light-front formalism. This form factor is used by David *et al.* [11] and reads:

$$F_{K^+}\left(Q^2\right) = \frac{a}{1+Q^2/\Lambda_1^2} + \frac{1-a}{\left(1+Q^2/\Lambda_2^2\right)^2} , \tag{320}$$

with $a = 0.398$, $\Lambda_1 = 0.642$ GeV, and $\Lambda_2 = 1.386$ GeV.
An alternative form proposed by [28] is:

$$F_{K^+}\left(Q^2\right) = \frac{1}{1+Q^2/\Lambda^2} , \tag{321}$$

with $\Lambda^2 = 0.61$ GeV^2.
Although K^0 is a neutral particle, the mass difference of the up and strange quark causes a nonzero form factor at finite Q^2. For the $(\gamma K^0 K^0)$ vertex we use a form factor derived within a vector meson dominance model [7]:

$$F_{K^0}\left(Q^2\right) = \frac{-1/3}{1+Q^2/m_\omega^2} + \frac{1/3}{1+Q^2/m_\phi^2} , \tag{322}$$

with $m_\omega = 781.9$MeV and $m_\phi = 1019.4$ MeV.

At the $(\gamma K^{*+} K^+)$, $(\gamma K_1 K^+)$ and $(\gamma K^{*0} K^0)$ vertices, a monopole transition form factor is assumed:

$$F\left(Q^2\right) = \frac{1}{1+Q^2/\Lambda^2} , \tag{323}$$

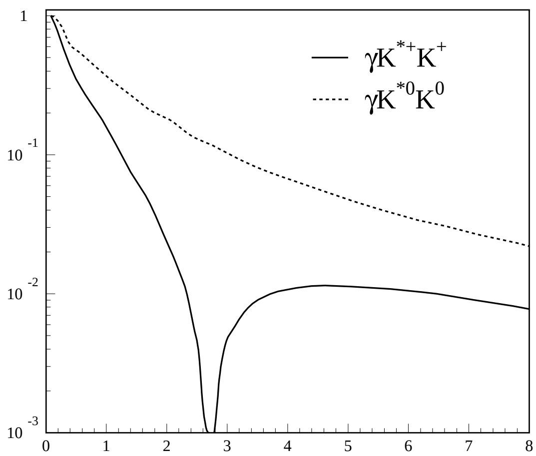

Figure 3: The vector meson transion form factors form [32]

with $\Lambda_{K^{*+}} = 0.95$ GeV, $\Lambda_{K_1} = 0.55$ GeV and $\Lambda_K^{*0} = m_\rho^2$.

An alternative for this transition form factors is given by [32]. We have no parameterization for it. The transition form factors are given in Fig. 3.

8.1.4 Real Photon Limit

For the sake of completeness, we report the real photon limits $(Q^2 = 0)$ of all the form factors used in our model:

$$\begin{array}{ccc} F_1^p = F_2^p = 1 \ , & F_1^{\Lambda,\Sigma^0} = 0 \ , & F_2^{\Lambda,\Sigma^0} = 1 \ , \\ F^{K,K^*,K_1} = 1 \ , & F^{K^0} = 0 \ , & F^{N^*} = F^{Y^*} = 1 \ . \end{array} \tag{324}$$

8.2 Hadronic Vertices

Until now, the hadronic vertex is treated as a point interaction. In order to account for the composed nature of this vertex, we can introduce hadronic form factors.

8.2.1 Gauge Invariance

Here again we have to pay attention to preserve gauge invariance. Introducing form factors at the resonance couplings causes no problems since each resonance amplitude is constructed in a gauge invariant way. Problems arise for the born terms. We arrive at the following amplitude for the born terms in the case of $p(\gamma, K^+)Y^0$ processes.

$$
\begin{aligned}
\mathcal{M}_{born} &= e g_{KYp} \overline{U}_Y \left[F_s(s) \gamma_5 \frac{\not{p} + \not{k} + M_p}{s - M_p^2} \left(\gamma^\mu + \frac{\kappa_p}{2M_p} i\sigma^{\mu\nu} k_\nu \right) \right. \\
&+ F_t(t) \gamma_5 \left(2p_K^\mu - k^\mu \right) \frac{1}{t - M_K^2} \\
&\left. + F_u(u) \frac{\kappa_Y}{2M_p} i\sigma^{\mu\nu} k_\nu \frac{\not{p}_Y - \not{k} + M_Y}{u - M_Y^2} \gamma_5 \right] U_p \varepsilon_\mu
\end{aligned} \tag{325}
$$

In each channel appears a unique form factor ($F_s(s), F_t(t), F_u(u)$) that depends on the corresponding Mandelstam variable. The two terms that violate current conservation are the electric term in the s-channel and the t-channel amplitude and it is clear that after the introduction of form factors makes that this two parts do not cancel any more in the gauge condition $k_\mu \mathcal{M}^\mu = 0$.

There are different way's to restore this gauge invariance. We will follow the approach of Haberzettl *et al.* [22]. The main philosophy of this technique is to introduce a new form factor $\widehat{F}$ that is equal for this two pieces, so that they keep on canceling in the gauge condition. Formally this goes as:

$$
\begin{aligned}
\mathcal{M}_{born} &= e g_{KYp} \overline{U}_Y \left[\gamma_5 \frac{\not{p} + \not{k} + M_p}{s - M_p^2} \left(\widehat{F} \gamma^\mu + F_s(s) \frac{\kappa_p}{2M_p} i\sigma^{\mu\nu} k_\nu \right) \right. \\
&+ \widehat{F} \gamma_5 \left(2p_K^\mu - k^\mu \right) \frac{1}{t - M_K^2} \\
&\left. + F_u(u) \frac{\kappa_Y}{2M_p} i\sigma^{\mu\nu} k_\nu \frac{\not{p}_Y - \not{k} + M_Y}{u - M_Y^2} \gamma_5 \right] U_p \varepsilon_\mu \\
&+ \mathcal{M}_{viol}
\end{aligned} \tag{326}
$$

where $\mathcal{M}_{viol}$ contains the gauge violating part:

$$
\begin{aligned}
\mathcal{M}_{viol} &= -e g_{KYp} \overline{U}_Y \gamma_5 \left[\frac{2p^\mu + \not{k} \gamma^\mu}{s - M_p^2} \left(\widehat{F} - F_s(s) \right) \right. \\
&\left. + \frac{2p_K^\mu}{t - M_K^2} \left(\widehat{F} - F_t(t) \right) \right] U_p \varepsilon_\mu \, .
\end{aligned} \tag{327}
$$

In fact, in Haberzettls approach, $\widehat{F}$ acts in the s-channel electric term only on the part $(\not{p} + M_p)$ and not on the $\not{k}$ of the propagator. For programming reasons we adopt this method but stress that gauge invariance also in this case is preserved. A contact term is now introduced in such a way that the amplitude of this contact term exactly absorbs this violating part:

$$
\overline{U}_Y (p_Y) T_c^\mu U_p (p) \varepsilon_\mu = -\mathcal{M}_{viol} \tag{328}
$$

and we end up with a amplitude that is gauge invariant after the introduction of hadronic form factors. Remark that the two form factors in the s-channel in Eq.(326) are hadronic form factors and that they have nothing to do with the Pauli and Dirac electromagnetic form factors of the previous section!

8.2.2 Form Factors

When all the external legs are on-shell, the form factors are only function of the Mandelstam variables:

$$\begin{aligned} F_s(s) &= f\left((p+k)^2, M_Y^2, M_K^2\right) , &(329)\\ F_u(u) &= f\left(M_p^2, (p_Y-k)^2, M_K^2\right) , &(330)\\ F_u(u) &= f\left(M_p^2, M_Y^2, (p_K-k)^2\right) , &(331)\end{aligned}$$

where $f\left(a^2, b^2, c^2\right)$ is a general meson-baryon form factor depending on the squared four-momenta of its three hadronic legs with the normalization $f(M_p^2, M_Y^2, M_K^2) = 1$. The parameterization of this function is according to Haberzettl

$$f\left(a^2, b^2, c^2\right) = \frac{\Lambda^4}{\Lambda^4 + \left(a^2 - M_p^2\right)^2 + (b^2 - M_Y^2)^2 + (c^2 - M_K^2)^2} . \qquad (332)$$

This general form reduces to

$$\begin{aligned} F_s(s) &= f\left(s, M_Y^2, M_K^2\right) &= \frac{\Lambda^4}{\Lambda^4+\left(s-M_p^2\right)^2} ,\\ F_u(u) &= f\left(M_p^2, u, M_K^2\right) &= \frac{\Lambda^4}{\Lambda^4+\left(u-M_Y^2\right)^2} ,\\ F_t(t) &= f\left(M_p^2, M_Y^2, t\right) &= \frac{\Lambda^4}{\Lambda^4+\left(t-M_K^2\right)^2} ,\end{aligned} \qquad (333)$$

For the from factor $\widehat{F}$ Haberzettl proposed a form like

$$\begin{aligned} \widehat{F} &= a_s F_s(s) + a_u F_u(u) + a_t F_t(t)\\ &= \widehat{F}(s,u,t) , \end{aligned} \qquad (334)$$

with the restriction that $a_s + a_u + a_t = 1$. In a first step, we will restrict ourself to the case $a_u = 0$ since $\widehat{F}$ acts only in the s- and t-channel.

Recently, Davidson and Workman [12] pointed out that the form of $\widehat{F}$ proposed by Haberzettl in Eq. (334) is not appropriate. In order to restore the gauge invariance, and additional contact term is introduced which identically cancels with the gauge breaking terms. The problem with the Haberzettl form is that the corresponding contact term is not "pool free", what it should be to be a contact term. An alternative, proposed by Workman *et al.* is:

$$\begin{aligned} \widehat{F} &= F_s(s) + F_t(t) - F_s(s)F_t(t)\\ &= \widehat{F}(s,t) , \end{aligned} \qquad (335)$$

As is clear form Eq.(332), beside the Mandelstam variables, the form factors also depends on a cutoff parameter Λ. We will choose two different cutoffs, one for the born terms and one for the resonance terms.

9 The Different Isospin Channels

9.1 Intermediate Particles

The different isospin channels in kaon production with their intermediate Born particles are:

$$\gamma + p \rightarrow \left\{ \begin{array}{l} p \\ K^+ \\ \Lambda^0 \\ \Sigma^0 \end{array} \right. \rightarrow K^+ + \Lambda^0 \tag{336}$$

$$\gamma + p \rightarrow \left\{ \begin{array}{l} p \\ K^+ \\ \Sigma^0 \\ \Lambda^0 \end{array} \right. \rightarrow K^+ + \Sigma^0 \tag{337}$$

$$\gamma + p \rightarrow \left\{ \begin{array}{l} p \\ (K^0) \\ \Sigma^+ \end{array} \right. \rightarrow K^0 + \Sigma^+ \tag{338}$$

$$\gamma + n \rightarrow \left\{ \begin{array}{l} n \\ (K^0) \\ \Lambda^0 \\ \Sigma^0 \end{array} \right. \rightarrow K^0 + \Lambda^0 \tag{339}$$

$$\gamma + n \rightarrow \left\{ \begin{array}{l} n \\ (K^0) \\ \Sigma^0 \\ \Lambda^0 \end{array} \right. \rightarrow K^0 + \Sigma^0 \tag{340}$$

$$\gamma + n \rightarrow \left\{ \begin{array}{l} n \\ K^+ \\ \Sigma^- \end{array} \right. \rightarrow K^+ + \Sigma^- \tag{341}$$

Remark that K^0, given between brackets, normally can not be an exchange particle since the photon can not couple to a neutral meson. Nevertheless, in the case of electroproduction, the form factor $F_{K^0}\left(Q^2\right)$ is not vanishing for finite Q^2 and this type of exchange can contribute to the process. See for an explicit form of the form factor Eq. (322).

9.2 Hadronic Form Factors

It is straight forward to extension of the hadronic form factor formalism, introduced in Sec. 8.2, to the other isospin channels. In the case of a $p(\gamma, K^0)\Sigma^+$ process there is no t-channel born term and the two gauge breaking terms are the electric couplings in the s- and u-channel. So, $\widehat{F}$ has to act on this two parts. For the $n(\gamma, K^0)Y^0$ reaction, there is no t-channel and since the baryons are all neutral, there are no electric couplings. Gauge invariance causes no problems. Finally, for a $n(\gamma, K^+)\Sigma^-$ process, $\widehat{F}$ has to act on the t-channel and electric part of the u-channel.

As mentioned above, the nonzero nature of the neutral electromagnetic form factors (F_n^1 and F_{K^0}) can allow electric couplings to this neutral currents at finite Q^2 (electroproduction). So, for neutral electric coupling we add the "normal" hadronic form factor corresponding to the channel of the current (F_s, F_t or F_u). Remark that the contribution of this terms will be small since the suppression of the electromagnetic form factor.

9.3 Magnetic Moments

The anomalous magnetic moments κ_x are given by:

Anomalous magnetic moment	Value
κ_p	1.793
κ_n	−1.913
κ_Λ	−0.613
κ_{Σ^+}	1.458
κ_{Σ^-}	−0.160
κ_{Σ^0}	0.790^a
$\lvert\kappa_{(\Sigma^0\Lambda)}\rvert$	1.610

(342)

The numeric values are from Ref.[34]. The hyperon Σ^0 has a very short lifetime ($7.4 \cdot 10^{-20}$s) and decays electromagnetically via $\Sigma^0 \to \Lambda + \gamma$. This decay occurs too fast to measure its magnetic moment. Therefor, we give here (see (a)) a quark model prediction. Remark that the Particle Data Group [34] collects the values for the magnetic moments μ_x of the particles. The relation with the anomalous magnetic moments κ_x given here is:

$$\begin{cases} \mu_x = (\kappa_x \pm 1) & \text{(charged particles)} \\ \mu_x = \kappa_x & \text{(neutral particles)} \end{cases} \tag{343}$$

Remark that all the values of μ_x in [34] are normalized to the nuclear magneton ($\mu_N = e\hbar/2M_p$) which is in agreement with our definition of the interaction Lagrangians in Sec. 3.

In the case of Σ^- production, the electric coupling becomes $-e\gamma^\mu$ whereas the magnetic coupling reads $(\mu_{\Sigma^-}+1)\frac{e}{2M_p}i\sigma^{\mu\nu}k_\nu$. Remark that the negative charge is only explicit contained in the electric term!

A point of confusion is the sign of $\kappa(\Sigma^0\Lambda)$ since this is experimentally not accessible. Using the convention of de Swart [13] based on (ideal) SU(3) symmetry and commonly used for the hadronic vertices, it turns out that:

$$g_{\Lambda KN} = -\frac{1}{\sqrt{3}}(3-2\alpha)\, g_{\pi NN} \;, \tag{344}$$

$$g_{\Sigma KN} = (2\alpha-1)\, g_{\pi NN} \;, \tag{345}$$

$$\mu\left(\Sigma^0\Lambda\right) = -\frac{\sqrt{3}}{2}\mu_n \;, \tag{346}$$

where $\alpha = D/(D+F)$ is the fraction of D-type coupling. Given that the SU(3) symmetry is broken at the level of 20%, and a value for $\alpha = 0.644$ [14] one arrive at:

$$\begin{aligned} -4.5 \leq \;& g_{\Lambda KN}/\sqrt{4\pi} \leq -3.0 \;, \\ 0.9 \leq \;& g_{\Sigma KN}/\sqrt{4\pi} \leq 1.3 \;, \end{aligned} \tag{347}$$

and

$$\mu\left(\Sigma^0\Lambda\right) = 1.65\;\mu_N \;. \tag{348}$$

Remark that in our model only the product $\left(\kappa(\Sigma^0\Lambda)\, g_{\Sigma KN}\right)$ is accessible as a model parameter and it turns out, using de Swarts convention, that this product has the opposite sign of $g_{\Lambda KN}$.

Contrary, using quark model predictions [36] the transition magnetic moment is given by:

$$\kappa(\Sigma^0\Lambda) = \frac{1}{\sqrt{3}}(\mu_d - \mu_u) \;, \tag{349}$$

$$= -1.63 \;. \tag{350}$$

To conclude, in our calculations we will use the boundary conditions of Eq. (347) for the main coupling constants, use the value given in the Table (342) for the transition magnetic moment with the (positive) sign given by de Swart convention.

9.4 Coupling Constant Relations

We are able to relate the coupling constants among the various isospin channels. The consequence of this is that the two Λ production processes can be described by one set of coupling constants and the four Σ production channels by an other set. So, the six isospin channels are determined

by two sets of coupling constants.
The free parameters of the model are (in most of the cases) a combination of a hadronic coupling constant and an electromagnetic coupling. In order to relate the free parameters of our model among the various isospin channels, we have to find relations for this two types of coupling. In this case, the hadronic vertices causes no problems since we can assume isospin symmetry for the strong force. The electromagnetic coupling constants are more difficult to relate and we have to relay on experimental results (with large errors) of the photocoupling helicity amplitudes.

9.4.1 Hadronic vertices

First, we define the isospin states for the relevant particles:

$$p = |I = \frac{1}{2},\ I_3 = \frac{1}{2}\rangle\ , \tag{351}$$
$$n = |I = \frac{1}{2},\ I_3 = -\frac{1}{2}\rangle\ , \tag{352}$$
$$K^+ = |I = \frac{1}{2},\ I_3 = \frac{1}{2}\rangle\ , \tag{353}$$
$$K^0 = |I = \frac{1}{2},\ I_3 = -\frac{1}{2}\rangle\ , \tag{354}$$
$$\Lambda = |I = 0,\ I_3 = 0\rangle\ , \tag{355}$$
$$\Sigma^+ = -\,|I = 1,\ I_3 = 1\rangle\ , \tag{356}$$
$$\Sigma^0 = |I = 1,\ I_3 = 0\rangle\ , \tag{357}$$
$$\Sigma^- = |I = 1,\ I_3 = -1\rangle\ . \tag{358}$$

Remark that the minus sign for Σ^+ is in agreement with the definition $Y^*_{l,m} = (-1)^m\, Y_{l,-m}$ since Σ^- is (by convention) chosen positive.
If the strong force that build up the hadronic vertices is invariant under isospin symmetry, one can relate the coupling constants in the different channels. This is done by constructing a Clebsch-Gordan coefficient for every coupling.

Λ coupling constants: For the coupling of a Λ hyperon to a nucleon and a kaon, one can construct coefficients as:

$$g_{\Lambda K^+ p} \longrightarrow \langle 0\,0,\ \frac{1}{2}\,\frac{1}{2}\,|\,\frac{1}{2}\,\frac{1}{2}\rangle = \frac{1}{\sqrt{2}}\ , \tag{359}$$
$$g_{\Lambda K^0 n} \longrightarrow \langle 0\,0,\ \frac{1}{2}\ -\frac{1}{2}\,|\,\frac{1}{2}\ -\frac{1}{2}\rangle = \frac{1}{\sqrt{2}}\ , \tag{360}$$

$$g^{\nu,t}_{\Lambda K^{*+} p} \longrightarrow \langle 0\,0,\ \frac{1}{2}\,\frac{1}{2}\,|\,\frac{1}{2}\,\frac{1}{2}\rangle = \frac{1}{\sqrt{2}}\ , \tag{361}$$
$$g^{\nu,t}_{\Lambda K^{*0} n} \longrightarrow \langle 0\,0,\ \frac{1}{2}\ -\frac{1}{2}\,|\,\frac{1}{2}\ -\frac{1}{2}\rangle = \frac{1}{\sqrt{2}}\ . \tag{362}$$

From the ratio of this Clebsch-Gordan coefficients one can conclude that:

$$g_{\Lambda K^+ p} = g_{\Lambda K^0 n} , \quad (363)$$
$$g^{\nu,t}_{\Lambda K^{*+} p} = g^{\nu,t}_{\Lambda K^{*0} n} . \quad (364)$$

Σ coupling constants: For the coupling of a Σ hyperon to a nucleon and a kaon, one can construct coefficients as:

$$g_{\Sigma^0 K^+ p} \rightarrow \langle 1\,0, \tfrac{1}{2}\,\tfrac{1}{2} \mid \tfrac{1}{2}\,\tfrac{1}{2}\rangle = -\frac{1}{\sqrt{3}} , \quad (365)$$
$$g_{\Sigma^0 K^0 n} \rightarrow \langle 1\,0, \tfrac{1}{2}\, -\tfrac{1}{2} \mid \tfrac{1}{2}\, -\tfrac{1}{2}\rangle = \frac{1}{\sqrt{3}} , \quad (366)$$
$$g_{\Sigma^+ K^0 p} \rightarrow -\langle 1\,1, \tfrac{1}{2}\, -\tfrac{1}{2} \mid \tfrac{1}{2}\,\tfrac{1}{2}\rangle = -\sqrt{\frac{2}{3}} , \quad (367)$$
$$g_{\Sigma^- K^+ n} \rightarrow \langle 1\,-1, \tfrac{1}{2}\,\tfrac{1}{2} \mid \tfrac{1}{2}\, -\tfrac{1}{2}\rangle = -\sqrt{\frac{2}{3}} , \quad (368)$$

and from the ratio's one can conclude again:

$$g_{\Sigma^0 K^+ p} = -\, g_{\Sigma^0 K^0 n} = \frac{g_{\Sigma^+ K^0 p}}{\sqrt{2}} = \frac{g_{\Sigma^- K^+ n}}{\sqrt{2}} \quad (369)$$

Remark that the same Clebsch-Gordan coefficients are obtained for the (YK^*N) vertices since K^* has the same isospin state as K. So, there holds:

$$g_{\Sigma^0 K^{*+} p} = -\, g_{\Sigma^0 K^{*0} n} = \frac{g_{\Sigma^+ K^{*0} p}}{\sqrt{2}} = \frac{g_{\Sigma^- K^{*+} n}}{\sqrt{2}} \quad (370)$$

Resonance coupling: The isospin states of the resonances are:

$$N^{*+,0} = |I = \tfrac{1}{2},\ I_3 = \pm\,\tfrac{1}{2}\rangle , \quad (371)$$
$$\Delta^{+,0} = |I = \tfrac{3}{2},\ I_3 = \pm\,\tfrac{1}{2}\rangle , \quad (372)$$
$$\Lambda^* = |I = 0,\ I_3 = 0\rangle , \quad (373)$$
$$\Sigma^{*0} = |I = 1,\ I_3 = 0\rangle , \quad (374)$$
$$\Sigma^{*\pm} = \mp\,|I = 1,\ I_3 = \pm\,1\rangle , \quad (375)$$

Where the $\pm$ for the Δ and N^* corresponds with the charged and neutral eigenstate of the resonance. Since N^*, Λ^* and Σ^* have the same isospin structure as their ground state, the relations in Eq. (363) and (369) remain valid for resonance coupling.

Only the Δ represents a new isospin state. Here for one can construct Clebsch-Gordan coefficients as:

$$g_{\Sigma^0 K^+ \Delta^+} \rightarrow \langle 1\,0, \tfrac{1}{2}\,\tfrac{1}{2} \mid \tfrac{3}{2}\,\tfrac{1}{2}\rangle = \sqrt{\frac{2}{3}} , \quad (376)$$

$$g_{\Sigma^+ K^0 \Delta^+} \quad \rightarrow \quad -\langle 1\,1,\ \frac{1}{2}\ -\frac{1}{2} \mid \frac{3}{2}\ \frac{1}{2}\rangle = -\sqrt{\frac{1}{3}}\,, \tag{377}$$

$$g_{\Sigma^0 K^0 \Delta^0} \quad \rightarrow \quad \langle 1\,0,\ \frac{1}{2}\ -\frac{1}{2} \mid \frac{3}{2}\ -\frac{1}{2}\rangle = \sqrt{\frac{2}{3}}\,, \tag{378}$$

$$g_{\Sigma^- K^+ \Delta^0} \quad \rightarrow \quad \langle 1\ -1,\ \frac{1}{2}\ \frac{1}{2} \mid \frac{3}{2}\ -\frac{1}{2}\rangle = \sqrt{\frac{1}{3}}\,. \tag{379}$$

The resulting relations for the coupling constants are:

$$g_{\Sigma^0 K^+ \Delta^+} \ = \ -\sqrt{2}\, g_{\Sigma^+ K^0 \Delta^+} \ = \ g_{\Sigma^0 K^0 \Delta^0} \ = \ \sqrt{2}\, g_{\Sigma^- K^+ \Delta^0} \tag{380}$$

9.4.2 Electromagnetic Vertices

Meson Transitions In K^0 production the vector meson exchanged in the t-channel is the K^{*0}. Hence, the transition moment $g_{K^* K\gamma}$ for the K^+ production must be replaced by the neutral transition moment. The transition moment is related to the decay width by (see Eq. (298):

$$\Gamma_{K^* \rightarrow K\gamma} \ = \ \frac{1}{3}\frac{g^2_{K^* K\gamma}}{4\pi M^2}|\vec{p}_K|^3\,, \tag{381}$$

$$= \ \frac{1}{24}\frac{g^2_{K^* K\gamma}}{4\pi M^2}\left[M_{K^*}\left(1-\frac{M_K^2}{M_{K^*}^2}\right)\right]^3\,. \tag{382}$$

since $|\vec{p}_K| = \frac{1}{2}M_{K^*}\left(1 - M_K^2/M_{K^*}^2\right)$ in the c.m. frame of the decay system. The measured decay widths are [34]:

$$\Gamma_{K^{*+} \rightarrow K^+\gamma} \ = \ 50 \pm 5\text{keV}\,, \tag{383}$$

$$\Gamma_{K^{*0} \rightarrow K^0\gamma} \ = \ 116 \pm 10\text{keV}\,, \tag{384}$$

Neglecting the small differences in the $K^{(*)+}$ and $K^{(*)0}$ masses compared to the errors of the decay widths, we end up with the relation:

$$\frac{g^2_{K^{*0}K^0\gamma}}{g^2_{K^{*+}K^+\gamma}} \ = \ \frac{\Gamma_{K^{*0}\rightarrow K^0\gamma}}{\Gamma_{K^{*+}\rightarrow K^+\gamma}}\,, \tag{385}$$

$$\left|\frac{g_{K^{*0}K^0\gamma}}{g_{K^{*+}K^+\gamma}}\right| \ = \ 1.52\,. \tag{386}$$

On this point, the sign is experimentally undetermined. According to a quark model prediction by Singer and Miller [23] it turns out that the K^{*0} amplitude has an opposite sign of the K^{*+} amplitude. So:

$$g_{K^{*0}K^0\gamma} = -1.52\, g_{K^{*+}K^+\gamma}\,. \tag{387}$$

Nucleon Transitions The nucleon transition moments are related to the photocoupling helicity amplitudes which are defined in Eq. (266), (266) and (268). This relations can easily be inverted to extract κ_{NN^*}. The helicity amplitudes are measured for a $(N^* \to p + \gamma)$ process but also for $(N^* \to n + \gamma)$. This measurements make it possible to relate the transition moments κ_{nN^*} with κ_{pN^*}. Apart from the small difference in the mass of the proton and the neutron, one can say that:

$$\text{spin } \tfrac{1}{2}: \qquad \frac{\kappa_{nN^*}}{\kappa_{pN^*}} = \frac{A^n_{1/2}}{A^p_{1/2}} \,, \tag{388}$$

$$\text{spin } \tfrac{3}{2}: \qquad \frac{\kappa^{(1)}_{nN^*}}{\kappa^{(1)}_{pN^*}} = \frac{\sqrt{3}A^n_{1/2} \pm A^n_{3/2}}{\sqrt{3}A^p_{1/2} \pm A^p_{3/2}} \,, \tag{389}$$

$$\frac{\kappa^{(2)}_{nN^*}}{\kappa^{(2)}_{pN^*}} = \frac{\sqrt{3}A^n_{1/2} - \frac{M_p}{M_{N^*}}A^n_{3/2}}{\sqrt{3}A^p_{1/2} - \frac{M_p}{M_{N^*}}A^p_{3/2}} \,. \tag{390}$$

The helicity amplitudes are collected in Table 4. Remark that for the resonance N8, predicted by Bennhold *et al.* [29], no experimental values are available.

The electromagnetic decay of the Δ is insensitive of the final isospin state. As a consequence of this property holds the equality $\kappa_{p\Delta^*} = \kappa_{n\Delta^*}$. From this one can conclude that Eq. (380) is valid for the complete coupling constant $G_{\Sigma K \Delta}$.

Hyperon Transitions: In principle, we can adopt the same procedure as in the previous paragraph to relate the hyperonic transition magnetic moments of the $(Y^*Y\gamma)$-vertices to helicity amplitudes. Unfortunately, there are no experimental predictions for these amplitudes of the processes $(Y^* \to \Sigma + \gamma)$ and $(Y^* \to \Lambda + \gamma)$.

So far, we assume that for the hyperonic decay into the different $\Sigma^{+,0,-}$ channels is independent of the final isospin (or charge) state. Remark that this same feature holds in the Δ^* decay case. So:

$$\kappa_{\Sigma^*\Sigma^0} = \kappa_{\Sigma^*\Sigma^+} = \kappa_{\Sigma^*\Sigma^-} \,. \tag{391}$$

10 Multipole Decomposition

10.1 General remarks

Given the reaction:

$$\gamma\left(1^-\right) + N\left(1/2^+\right) \to K\left(0^-\right) + Y\left(1/2^+\right) \,, \tag{392}$$

where the spin and parity of the corresponding particles is denoted between brackets. Since the reaction conserves the total angular momentum, we can write:

$$\vec{J} = \left(\vec{1}\right)_\gamma + \left(\frac{\vec{1}}{2}\right)_N + \vec{l}_i = \left(\frac{\vec{1}}{2}\right)_Y + \vec{l}_f \,. \tag{393}$$

L	l_i	Photon multipole	J	$\mathcal{P}$	l_f	Kaon multipole	Contributing N^* or Δ^*
1	0 or 2	E1	1/2	-	0	E_{0+}	S_{11}
			3/2	-	2	E_{2-}	D_{13}
	1	M1	1/2	+	1	M_{1-}	P_{11}
			3/2	+	1	M_{1+}	P_{13}
2	1 or 3	E2	3/2	+	1	E_{1+}	P_{13}
			5/2	+	3	E_{3-}	F_{15}
	2	M2	3/2	-	2	M_{2-}	D_{13}
			5/2	-	2	M_{2+}	D_{15}
3	2 or 4	E3	5/2	-	2	E_{2+}	D_{15}
			7/2	-	4	E_{4-}	
	3	M3	5/2	+	3	M_{3-}	F_{15}
			7/2	+	3	M_{3+}	
4	3 or 5	E4	7/2	+	3	E_{3+}	
			9/2	+	5	E_{5-}	
	4	M4	7/2	-	4	M_{4-}	
			9/2	-	4	M_{4+}	
...							

Table 7: Connection of the photon and kaon multipole decomposition and the contributions of the N^* and Δ^* resonances.

Herein are $\vec{l}_i$ and $\vec{l}_f$ the relative orbital angular momenta of the incoming and outgoing particles. For a given J the relation is *a priori* fulfilled for 8 different combinations of the spin projections (2 for the photon, 2 for the nucleon and 2 for the hyperon).

Beside the total angular momentum, also the total parity is conserved in the reaction:

$$\mathcal{P} = \mathcal{P}_\gamma \times \mathcal{P}_N \times (-1)^{l_i} = \mathcal{P}_K \times \mathcal{P}_Y \times (-1)^{l_f} \ . \tag{394}$$

From this one can conclude that the relative angular momenta have to fulfill the relation:

$$(-1)^{l_i} = (-1)^{l_f} \ . \tag{395}$$

This additional restriction of parity conservation reduce the 8 independent numbers to 4. So, to describe the reaction at a fixed J, 4 independent amplitudes have to be determined.

This independent amplitudes can be labeled when one relay on the multiplicity of the photon. Combining the photon spin $\vec{1}$ with the relative angular momentum l_i of the incoming channel, gives three states for $\vec{L}$ ($|L| = l_i, l_i \pm 1$). These correspond to two different parity states. The $|L| = l_i \pm 1$ states have parity $\mathcal{P} = (-1)^L$ and are called the *electric multipoles* EL. On the other hand, the $|L| = l_i$ states have parity $\mathcal{P} = (-1)^{L+1}$ and are called the *magnetic multipoles* ML. To determine the total system, the photon multipoles have to be combined with the spin of the nucleon which give the possible angular momentum configurations.

Until now, the labeling was done for the incoming channel. The multipole decomposition can also be made for the outgoing channel. This is results in four different multipoles E_{l_f+}, E_{l_f-}, M_{l_f+} and M_{l_f-}, where the sign is determined by the relation $J = l_f \pm \frac{1}{2}$. These relations and the connection with the contributing resonances in the s-channels are summarized in Table 7.

Since in our formalism, we do not have explicit access to the amplitudes of the process, we can not immediately decompose the amplitude in his multipoles. To do this, we first have to define the helicity amplitudes which made it possible to relate the observables (cross sections and polarization asymmetries) and the multipoles.

10.2 Helicity amplitudes

The helicity amplitudes are defined as:

$$A_{\mu\lambda} = \overline{U}(p_Y, \lambda_Y)\, \tilde{T}(\lambda_\gamma)\, U(p, \lambda_p) \ , \tag{396}$$

where λ_p, λ_Y and λ_γ are the helicity of the nucleon, the hyperon and the photon, $\tilde{T}(\lambda_\gamma)$ the transition operator as a function of the photon helicity and

$$\lambda = \lambda_\gamma - \lambda_p \ , \tag{397}$$

$$\mu = -\lambda_Y \ , \tag{398}$$

the helicity of the initial and final state. Parity relations connect some of the possible helicity combinations. There are four of them independent. According to Ref.[31] we define

$$N \equiv A_{1/2,1/2} \ , \tag{399}$$

$$S_1 \equiv A_{1/2,3/2} \ , \tag{400}$$

$$S_2 \equiv A_{-1/2,1/2} \ , \tag{401}$$

$$D \equiv A_{-1/2,3/2} \ , \tag{402}$$

where N and D are the non-flip and the double-flip amplitudes and S_1 and S_2 are the single-flip amplitudes with the initial photon and nucleon spin parallel and anti parallel. The connection with the other helicity combinations is given by:

$$A_{-\mu-\lambda} = -e^{i(\lambda-\mu)\pi} A_{\mu\lambda} \ . \tag{403}$$

All the observables can now be expressed as a function of this four independent amplitudes [3], [31]. This relations are summarized in Table 8. Remark that there is a change in sign for the beam-target and the beam-recoil asymmetries. This is due to the fact that Baker *et al.* have defined their positive hadron polarization in an opposite way as we did in Table 1.

Observable	Helicity representation
$d\sigma/d\Omega$	$\lvert N\rvert^2 + \lvert S_1\rvert^2 + \lvert S_2\rvert^2 + \lvert D\rvert^2$
$\Sigma d\sigma/d\Omega$	$2\,\mathrm{Re}\,(S_1 S_2^* - ND^*)$
$T d\sigma/d\Omega$	$2\,\mathrm{Im}\,(S_1 N^* - S_2 D^*)$
$P d\sigma/d\Omega$	$2\,\mathrm{Im}\,(S_2 N^* - S_1 D^*)$
$E d\sigma/d\Omega$	$\lvert D\rvert^2 + \lvert S_1\rvert^2 - \lvert S_2\rvert^2 - \lvert N\rvert^2$
$F d\sigma/d\Omega$	$-2\,\mathrm{Re}\,(S_2 D^* + S_1 N^*)$
$G d\sigma/d\Omega$	$2\,\mathrm{Im}\,(S_1 S_2^* + ND^*)$
$H d\sigma/d\Omega$	$2\,\mathrm{Im}\,(S_1 D^* + S_2 N^*)$
$O_x d\sigma/d\Omega$	$2\,\mathrm{Im}\,(S_2 D^* + S_1 N^*)$
$O_z d\sigma/d\Omega$	$2\,\mathrm{Im}\,(S_2 S_1^* + ND^*)$
$C_x d\sigma/d\Omega$	$2\,\mathrm{Re}\,(S_2 N^* + S_1 D^*)$
$C_z d\sigma/d\Omega$	$\lvert N\rvert^2 + \lvert S_1\rvert^2 - \lvert S_2\rvert^2 - \lvert D\rvert^2$
$T_x d\sigma/d\Omega$	$2\,\mathrm{Re}\,(S_1 S_2^* + ND^*)$
$T_z d\sigma/d\Omega$	$2\,\mathrm{Re}\,(S_1 N^* - S_2 D^*)$
$L_x d\sigma/d\Omega$	$2\,\mathrm{Re}\,(S_2 N^* - S_1 D^*)$
$L_z d\sigma/d\Omega$	$\lvert S_1\rvert^2 + \lvert S_2\rvert^2 - \lvert N\rvert^2 - \lvert D\rvert^2$

Table 8: Relations between the observables and the helicity amplitudes. Expressions are from Ref.[3]

$P_0(x)$	=	1
$P_1(x)$	=	x
$P_2(x)$	=	$\frac{1}{2}(3x^2-1)$
$P_3(x)$	=	$\frac{1}{2}(5x^3-3x)$
$P_4(x)$	=	$\frac{1}{8}(35x^4-30x^2+3)$
$P_5(x)$	=	$\frac{1}{8}(63x^5-70x^3+15x)$
$P_6(x)$	=	$\frac{1}{16}(231x^6-315x^4+105x^2-5)$
$P_7(x)$	=	$\frac{1}{16}(429x^7-693x^5+315x^3-35x)$
$P_8(x)$	=	$\frac{1}{128}(6435x^8-12012x^6+6930x^4-1260x^2+35)$
...		

Table 9: Legendre polynomials

When the four helicity amplitudes are determined from the observables, this quantities can be related to the multipoles of the final state:

$$\begin{aligned}
N(\omega,\theta) &= \frac{1}{\sqrt{2}}\cos\frac{\theta}{2}\sum_{l=0}^{\infty}\left[(l+2)E_{l+}+lM_{l+}+lE_{(l+1)-}-(l+2)M_{(l+1)-}\right]\left(P_l'-P_{l+1}'\right)\ ,\\
S_1(\omega,\theta) &= \frac{1}{\sqrt{2}}\cos\frac{\theta}{2}\sin\theta\sum_{l=0}^{\infty}\left[E_{l+}-M_{l+}-E_{(l+1)-}-M_{(l+1)-}\right]\left(P_l''-P_{l+1}''\right)\ ,\\
S_2(\omega,\theta) &= \frac{1}{\sqrt{2}}\sin\frac{\theta}{2}\sum_{l=0}^{\infty}\left[(l+2)E_{l+}+lM_{l+}-lE_{(l+1)-}+(l+2)M_{(l+1)-}\right]\left(P_l'+P_{l+1}'\right)\ ,\\
D(\omega,\theta) &= \frac{1}{\sqrt{2}}\sin\frac{\theta}{2}\sin\theta\sum_{l=0}^{\infty}\left[E_{l+}-M_{l+}+E_{(l+1)-}+M_{(l+1)-}\right]\left(P_l''+P_{l+1}''\right)\ ,
\end{aligned} \tag{404}$$

where P_l' are the derivatives of the of the Legendre polynomials. Inverting the relations of Table 8 and Eq. (404) leads to the multipoles of the process.

11 Legendre Decomposition

Since the Legendre polynomials form a complete set, we can decompose the differential cross section of the strangeness production process in:

$$\frac{d\sigma}{d\Omega}(\omega_{lab},\cos\theta)=\frac{\sigma_{tot}}{4\pi}\left[1+\sum_{l=1}^{\infty}C_l(\omega_{lab})P_l(\cos\theta)\right]\ , \tag{405}$$

where $C_l \equiv a_l/a_0$ and the Legendre coefficients a_l are defined as:

$$a_l=\frac{2l+1}{2}\int_{-1}^{+1}\frac{d\sigma}{d\Omega}(t)\,P_l(t)\,dt\ . \tag{406}$$

Resonance set	Hadronic form factor	χ^2
Born N4 N6 N7	soft	3.83
Born N4 N6 N7	hard	9.56
Born N4 N6 N7 N8	soft	4.29
Born N4 N6 N7 N8	hard	6.77
Born N4 N6 N7 L1 L3	hard	3.09
Born N4 N6 N8 L1 L3	hard	4.73
Born N4 N6 N7 N8 L1 L3	hard	3.17

Table 10: Results for the Λ production process

The Legendre polynomials are given in Table 9. In this way, it is easy to prove that

$$a_0 = \frac{\sigma_{tot}}{4\pi} \, . \tag{407}$$

Remark that the coefficients a_l are energy dependent quantities. Only the angular distribution is incorporated in this decomposition.

We can construct a relation between the Legendre decomposition of the cross section and the multipole decomposition of the transition amplitude. This has to be worked out!!!!

12 Fitting Strategy

We stress that the approach followed in this work starts from effective Lagrangians. So, all the coupling constants are in principle free parameters. They must be determined by a fit to the experimental observables.

A possible fitting method, explained in [6], calculates the weighted least-squares function χ^2:

$$\chi^2 = \sum_i \frac{\left[X_i - Y_i\left(a_1, \ldots a_n\right)\right]^2}{\sigma^2_{X_i}} \, , \tag{408}$$

where X_i represent the experimental observables, $\sigma^2_{X_i}$ are their standard deviations and $Y_i\left(a_1, \ldots a_n\right)$ are the theoretical predictions with a's being the parameters of the theory. By minimizing this χ^2, a best set of parameters is achieved.

13 Results

The results for the reaction $p(\gamma, K^+)\Lambda$ are summarized in Table 10.

14 Regge Theory

To implement the Regge theory in our approach we have to replace the propagator [21]. In the low t domain, it is appropriate to Reggeize the t-channel propagator. In the low u domain, the u-channel propagators can be Reggeized.

14.1 t-channel Reggeization

The t-channel propagators are replaced according to:

$$\frac{1}{t - M_K^2} \Rightarrow \left(\frac{s}{s_0}\right)^{\alpha_K(t)} \frac{\pi \alpha'_K}{\sin\left(\pi \alpha_K(t)\right)} \frac{1 + \zeta_K e^{-i\pi\alpha_K(t)}}{2} \frac{1}{\Gamma\left(1 + \alpha_K(t)\right)} , \tag{409}$$

$$\frac{1}{t - M_{K^*}^2} \Rightarrow \left(\frac{s}{s_0}\right)^{\alpha_{K^*}(t)-1} \frac{\pi \alpha'_{K^*}}{\sin\left(\pi \alpha_{K^*}(t)\right)} \frac{1 + \zeta_{K^*} e^{-i\pi\alpha_{K^*}(t)}}{2} \frac{1}{\Gamma\left(\alpha_{K^*}(t)\right)} . \tag{410}$$

Herein is s_0 a scaling factor which is set to 1 GeV. ζ is the signature of the trajectory. Since a trajectory decouples in an even and odd trajectory, two amplitudes have to be taken into account. This couple of trajectories can be degenerated or non-degenerated. The corresponding amplitude can be obtained by adding or subtracting this two trajectories with opposite signature $\zeta = \pm 1$. This results in different overall phases:

$$\frac{1 + e^{-i\pi\alpha(t)}}{2} \pm \frac{1 - e^{-i\pi\alpha(t)}}{2} = \begin{cases} 1 & \text{constant phase,} \\ e^{-i\pi\alpha(t)} & \text{rotating phase.} \end{cases} \tag{411}$$

The standard trajectories are given by:

$$\alpha_K(t) = 0.7\left(t - m_K^2\right) , \tag{412}$$

$$\alpha_{K^*}(t) = 1 + 0.83\left(t - m_{K^*}^2\right) . \tag{413}$$

In order to restore gauge invariance, which is broken by the introduction of the Regge propagator, we multiply the electric part of the s-channel Born term with the same factor as we did for the t-channel, e.g. $\mathcal{P}^K_{Regge} \cdot \left(t - m_K^2\right)$. So we obtain for the amplitude $\mathcal{M}$:

$$\mathcal{M}\left(K^+, Y\right) = \mathcal{M}_t + \mathcal{M}_s \cdot \mathcal{P}^K_{Regge} \cdot \left(t - m_K^2\right) . \tag{414}$$

Remark that $\mathcal{M}_s$ only contains the electric part. The magnetic transition term is not included in order to respect duality.

14.2 u-channel Reggeization

The u-channel propagators are replaced according to:

$$\frac{1}{u - M_Y^2} \quad \Rightarrow \quad \left(\frac{s}{s_0}\right)^{\alpha_Y(u)-\frac{1}{2}} \frac{\pi\alpha'_Y}{\sin\left(\pi\left(\alpha_Y(u)-\frac{1}{2}\right)\right)} \frac{1+\zeta_Y e^{-i\pi\left(\alpha_Y(u)-\frac{1}{2}\right)}}{2} \frac{1}{\Gamma\left(\frac{1}{2}+\alpha_Y(u)\right)} , \tag{415}$$

$$\frac{1}{u - M_{Y^*}^2} \quad \Rightarrow \quad \left(\frac{s}{s_0}\right)^{\alpha_{Y^*}(u)-\frac{3}{2}} \frac{\pi\alpha'_{Y^*}}{\sin\left(\pi\left(\alpha_{Y^*}(u)-\frac{3}{2}\right)\right)} \frac{1+\zeta_{Y^*} e^{-i\pi\left(\alpha_{Y^*}(u)-\frac{3}{2}\right)}}{2} \frac{1}{\Gamma\left(-\frac{1}{2}+\alpha_{Y^*}(u)\right)} , \tag{416}$$

where Y represents Λ^0 or Σ^0 (spin 1/2) and Y^* is a spin 3/2 hyperon resonance. In practice, the first materialization of this trajectory is the $\Sigma^*(1385)$. Remark that also here, the trajectories are degenerated and that the signature results in a constant or a rotating phase. The trajectories are parameterized as:

$$\alpha_\Lambda(u) = -0.6 + 0.90\,u , \tag{417}$$

$$\alpha_\Sigma(u) = -0.6 + 0.85\,u , \tag{418}$$

$$\alpha_{\Sigma^*}(u) = -0.3 + 0.90\,u . \tag{419}$$

Since the first materialization of the Σ^* trajectory is a $3/2^+$ particle, we have to include, apart from the u-channel Born terms, a u-channel spin $3/2^+$ exchange diagram. This diagram is determined by two coupling constants, assuming that no off-shell parameters are contributing, who read:

$$G_{\Sigma^*}^{(1)} = \frac{e f_{KY\Sigma^*}}{4\pi} \kappa_{Y\Sigma^*}^{(1)} , \tag{420}$$

$$G_{\Sigma^*}^{(2)} = \frac{e f_{KY\Sigma^*}}{4\pi} \kappa_{Y\Sigma^*}^{(2)} . \tag{421}$$

No experimental values are present for these coupling constants. We determine them by a fit to the data.

14.3 Duality corrections

When Regge theory is extrapolated into the resonance region, a duality correction has to be taken into account. Note that a factor 1/2 is omitted for the simplicity of notation. This correction is only relevant in the resonance region, since it disappear for higher s values and it reads:

$$\begin{aligned} &1 + \zeta_t\, e^{-i\pi\alpha(t)} \rightarrow \\ &\qquad 1 + \zeta_t\, e^{-i\pi\alpha(t)} + \zeta_t\zeta_s\, \sin\pi\alpha(t) \left[\frac{1+\zeta_s\, e^{i\pi\alpha(s)}}{\sin\pi\alpha(s)}\right] , \end{aligned} \tag{422}$$

where ζ_t (ζ_s) is the signature of the t- (s-) channel trajectory. The correction factor can be rewritten as:

$$1 + \zeta_t\, e^{-i\pi\alpha(t)} - i\,\zeta_t\zeta_s\, \sin\pi\alpha(t) \left[\frac{e^{i\pi\alpha_R(s)} - \zeta_s\, e^{-\pi\alpha_I(s)}}{\cosh\pi\alpha_I(s) - \zeta_s\, \cos\pi\alpha_R(s)}\right] , \tag{423}$$

where $\alpha = \alpha_R + i\alpha_I$. Separated in a real and imaginary part, this phase to the Regge propagator becomes:

$$1 + \zeta_t \cos \pi\alpha(t) + \zeta_t \zeta_s \sin \pi\alpha(t) \frac{\sin \pi\alpha_R(s)}{\cosh \pi\alpha_I(s) - \zeta_s \cos \pi\alpha_R(s)} + i\left(-\zeta_t \sin \pi\alpha(t) - \zeta_t \zeta_s \sin \pi\alpha(t) \frac{\cos \pi\alpha_R(s) - \zeta_s e^{-\pi\alpha_I(s)}}{\cosh \pi\alpha_I(s) - \zeta_s \cos \pi\alpha_R(s)}\right) . \quad (424)$$

Note that the first and the second term in the real part and the first term in the imaginary part are the ordinary Regge phase.

Remark that to ensure gauge invariance we have introduced the electric part of the s-channel and reggeized the corresponding propagator. In order to avoid double counting, we have to subtract the nucleon pole term in the duality correction term. The detailed form for this expression is:

$$\zeta_t \zeta_s \sin \pi\alpha(t) \left[\frac{\pi\alpha_R'\left(s - M_p^2\right)}{\cosh \pi\alpha_I(s) - \zeta_s\left(1 - \frac{1}{2}\left[\pi\alpha_R'\left(s - M_p^2\right)\right]^2\right)} - i\frac{1 - \frac{1}{2}\left[\pi\alpha_R'\left(s - M_p^2\right)\right]^2 - \zeta_s e^{-\pi\alpha_I(s)}}{\cosh \pi\alpha_I(s) - \zeta_s\left(1 - \frac{1}{2}\left[\pi\alpha_R'\left(s - M_p^2\right)\right]^2\right)}\right] . \quad (425)$$

An unexpected consequence of this subtraction can be observed in the imaginary part of panel (b). This imaginary part receives for large $\alpha_R(s)$ and small $\alpha_I(s)$ an offset. This behavior reflects the coupling of the different materialization of the trajectory and indicates that if one pole is removed, this has implications for the entire trajectory. Drawback of this offset is the loss of the correct Breit-Wigner behavior produced by the duality correction term. Therefore, we have opted to correct for this unphysical effect by adding the term:

$$\zeta_s \frac{\pi\alpha_R'\left(s - M_p^2\right)}{e^{\pi\alpha_I(s)} + \zeta_s\left(\pi\alpha_R'\left(s - M_p^2\right)\right)^2} . \quad (426)$$

The two t-channel trajectories for the kaon and the K^* are degenerated and so far, we have chosen the rotating phase. Since the sign difference in ζ_t and the additional sign for the rotating phase, the duality corrected Regge phase becomes:

$$\cos \pi\alpha(t) + \sum_{\alpha(s)} \zeta_s \sin \pi\alpha(t) \frac{\sin \pi\alpha_R(s)}{\cosh \pi\alpha_I(s) - \zeta_s \cos \pi\alpha_R(s)} + i\left(-\sin \pi\alpha(t) - \sum_{\alpha(s)} \zeta_s \sin \pi\alpha(t) \frac{\cos \pi\alpha_R(s) - \zeta_s e^{-\pi\alpha_I(s)}}{\cosh \pi\alpha_I(s) - \zeta_s \cos \pi\alpha_R(s)}\right) , \quad (427)$$

where we have taken into account multiple s-channel trajectories. For $\alpha(t)$ we have used the expressions given above (see α_K and α_{K^*}). For the s-channel trajectory, we disentangle between the nucleon and the delta trajectories. The nucleon trajectories are parametrized as:

$$\begin{aligned}
\alpha_R(s)\left[N^*1/2^+\right] &= \alpha_N(s) - \frac{1}{2} = \alpha_N'\left(s - M_p^2\right) , && (428)\\
\alpha_R(s)\left[N^*1/2^-\right] &= \alpha_N(s) - \frac{1}{2} = \alpha_N'\left(s - M_{1675}^2\right) , && (429)\\
\alpha_R(s)\left[N^*3/2^+\right] &= \alpha_N(s) - \frac{3}{2} = \alpha_N'\left(s - M_{1720}^2\right) , && (430)\\
\alpha_R(s)\left[N^*3/2^-\right] &= \alpha_N(s) - \frac{3}{2} = \alpha_N'\left(s - M_{1520}^2\right) , && (431)\\
& && (432)
\end{aligned}$$

with the slope $\alpha_N' = 0.97\,\mathrm{GeV}^{-2}$ the same for all the trajectories. The imaginary part of the nucleon trajectories is parametrized as:

$$\begin{aligned}
\alpha_I(s)[N^*] &= 0.164 + 0.3\,(s - 2.31) ,\\
&= -0.529 + 0.3\,s , && (433)
\end{aligned}$$

and is identical for all of them. For the Δ^*'s we obtain:

$$\begin{aligned}
\alpha_R(s)\left[\Delta^*1/2^+\right] &= \alpha_\Delta(s) - \frac{1}{2} = \alpha_\Delta'\left(s - M_{1905}^2\right) , && (434)\\
\alpha_R(s)\left[\Delta^*1/2^-\right] &= \alpha_\Delta(s) - \frac{1}{2} = \alpha_\Delta'\left(s - M_{1930}^2\right) , && (435)\\
\alpha_R(s)\left[\Delta^*3/2^+\right] &= \alpha_\Delta(s) - \frac{3}{2} = \alpha_\Delta'\left(s - M_{1230}^2\right) , && (436)\\
\alpha_R(s)\left[\Delta^*3/2^-\right] &= \alpha_\Delta(s) - \frac{3}{2} = \alpha_\Delta'\left(s - M_{1700}^2\right) , && (437)\\
& && (438)
\end{aligned}$$

with the slope $\alpha_\Delta' = 0.92\,\mathrm{GeV}^{-2}$. For the imaginary part we use:

$$\alpha_I(s)[\Delta^*] = -0.115 + 0.173\,s , \qquad (439)$$

identical for all Δ^* trajectories.

So far, we have no optimal set for the relative signs of the s-channel trajectories (analogous to the constant or rotating phase in the t-channel case). Nevertheless, they are important in the final result! We have to check this out!!!

14.4 Other tricks to extrapolate towards the resonance region

- In the normal Regge theory, the energy dependence of the cross section goes as $s^{\alpha(t)}$. There is a suggestion to replace this recipe according to:

$$s^{\alpha(t)} \rightarrow \left(\frac{s-u}{2}\right)^{\alpha(t)} . \qquad (440)$$

This is motivated by the fact that for high s and low t, which is the Regge limit, $-u \to s$. Our numerical calculations did not prefer this option. The cross sections turn out to be far to huge!

- In [21] the suggestion is made to use saturating trajectories when one goes to higher t values. A parametrization of the saturating trajectory can be:

$$\begin{aligned} \alpha(t) &= \alpha_0 + \alpha' \, t & ; t > 0 \\ \alpha(t) &= (\alpha_0 + 1) \, e^{\frac{\alpha'}{\alpha_0 + 1} t} - 1 & ; t < 0 \end{aligned} \tag{441}$$

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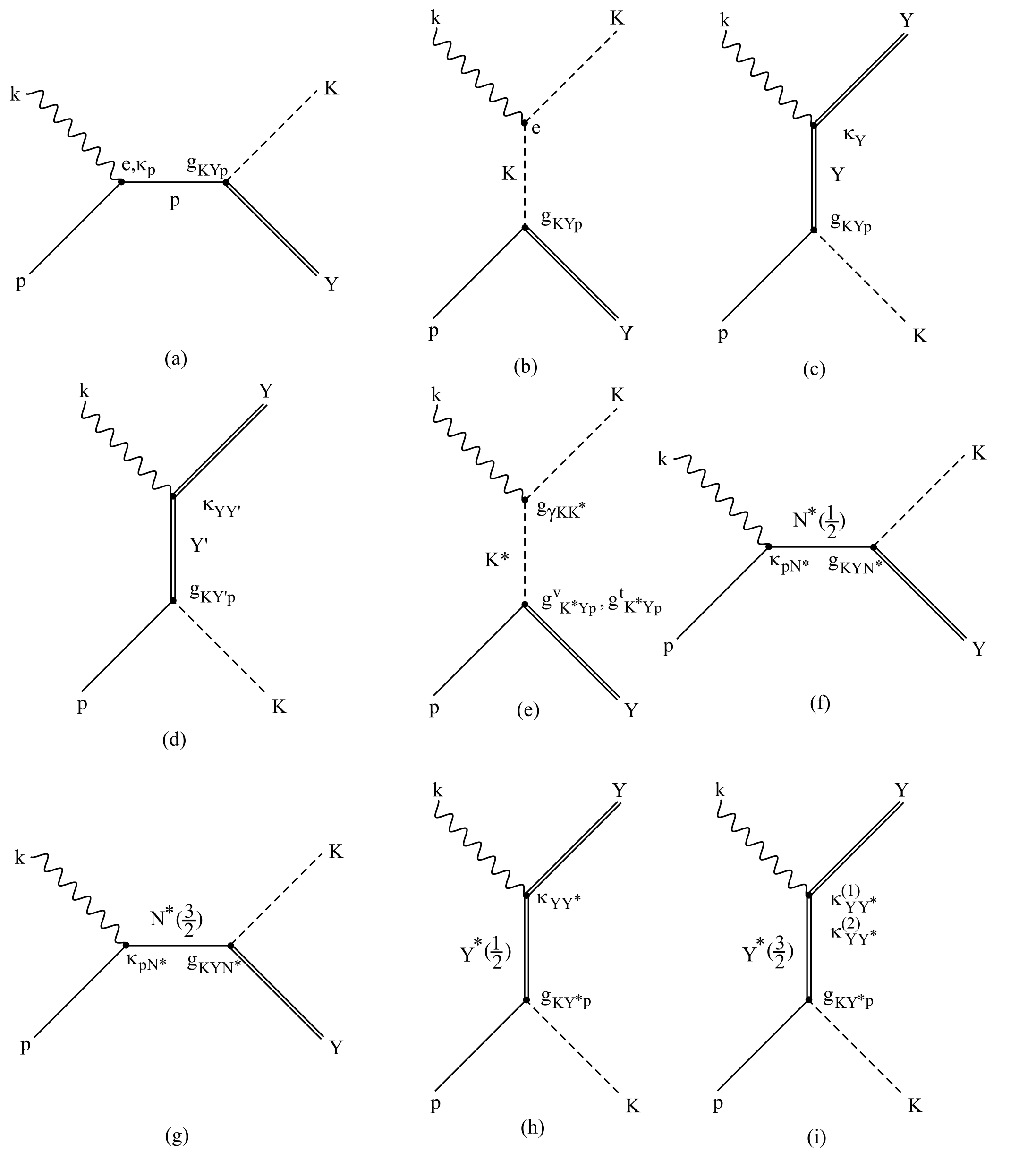

Figure 4: The different diagrams that occur in the photoproduction of a kaon on a proton. (a), (b) and (c) are the Born terms in the s-, t- and u-channel. In (d) is Y′ the other hyperon that is exchanged. In (e), a vector meson is exchanged in the t-channel. In (f) and (g) nucleon resonances in the s-channel are the intermediate particles whereas in (h) and (i) the exchanged particles are hyperon resonances in the u-channel.

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Notation	Particle	(l) J^π	Mass (MeV)	Width (MeV)
	p	$1/2^+$	938.272	
	n	$1/2^+$	939.565	
	K^+	0^-	493.677	
	K^0	0^-	497.672	
	Λ	$1/2^+$	1115.68	
	Σ^0	$1/2^+$	1192.64	
	Σ^+	$1/2^+$	1189.37	
	Σ^-	$1/2^+$	1197.45	
K^{*+}	$K^*(892)^+$	1^-	891.66	50.8
K^{*0}	$K^*(892)^0$	1^-	896.10	50.7
K1	$K_1(1270)$	1^+	1273.0	90.0
N1 - P_{11}	N (1440)	(1) $1/2^+$	1440	350
N2 - D_{13}	N (1520)	(2) $3/2^-$	1520	120
N3 - S_{11}	N (1535)	(0) $1/2^-$	1535	150
N4 - S_{11}	N (1650)	(0) $1/2^-$	1650	150
N5 - D_{13}	N (1700)	(2) $3/2^-$	1700	100
N6 - P_{11}	N (1710)	(1) $1/2^+$	1710	100
N7 - P_{13}	N (1720)	(1) $3/2^+$	1720	150
N8 - D_{13}	N (1895)	(2) $3/2^-$	1895	350
N9 - D_{15}	N (1675)	(2) $5/2^-$	1670	150
N10 - F_{15}	N (1680)	(3) $5/2^+$	1680	130
L1 - S_{01}	Λ (1405)	(0) $1/2^-$	1406	50
L2 - P_{01}	Λ (1600)	(1) $1/2^+$	1600	150
L3 - S_{01}	Λ (1670)	(0) $1/2^-$	1670	35
L4 - S_{01}	Λ (1800)	(0) $1/2^-$	1800	300
L5 - P_{01}	Λ (1810)	(1) $1/2^+$	1810	150
L6 - D_{03}	Λ (1520)	(2) $3/2^-$	1520	15.6
L7 - D_{03}	Λ (1690)	(2) $3/2^-$	1690	60
L8 - P_{03}	Λ (1890)	(1) $3/2^+$	1890	100
S1 - P_{11}	Σ (1660)	(1) $1/2^+$	1660	100
S2 - S_{11}	Σ (1750)	(0) $1/2^-$	1750	90
S3 - D_{13}	Σ (1670)	(2) $3/2^-$	1670	60
D1 - S_{31}	Δ (1620)	(0) $1/2^-$	1620	150
D2 - S_{31}	Δ (1900)	(0) $1/2^-$	1900	200
D3 - P_{31}	Δ (1910)	(1) $1/2^+$	1910	250
D4 - P_{33}	Δ (1232)	(1) $3/2^+$	1232	120
D5 - P_{33}	Δ (1600)	(1) $3/2^+$	1600	350
D6 - D_{33}	Δ (1700)	(2) $3/2^-$	1700	300
D7 - P_{33}	Δ (1920)	(1) $3/2^+$	1920	200

Table 11: Particles that can play a role in the photoproduction process of strangeness. The numeric values are from Ref.[34]